# ALGEBRA 1 LEAP 2025 REVIEW

Work through the packet and turn back in when we return from break.

KENNEDY & VIDRINE CLASSES

# Chapter 3
# Introduction to Algebra

Standards covered: A.SSE.A1, A.SSE.2, A.SSE.B.3

## 3.1 Algebra Vocabulary (DOK 1)

| Vocabulary Word | Example | Definition |
|---|---|---|
| **variable** | $4x$<br>(The variable is $x$.) | a letter that can be replaced by a number |
| **coefficient** | $4x$<br>(The coefficient is 4.) | a number multiplied by a variable or variables |
| **term** | $5x^2 + x - 2$<br>($5x^2$, $x$, and $-2$ are terms.) | numbers or variables separated by + or − signs |
| **constant** | $5x + 2y + 4$<br>(The constant is 4.) | a term that does not have a variable |
| **degree** | $4x^2 + 3x - 2$<br>(The degree is 2.) | the largest power of a variable in an expression |
| **leading coefficient** | $4x^2 + 3x - 2$<br>(The leading coefficient is 4.) | the number multiplied by the term with the highest power |
| **expression** | $5x - 2y + 6$ | can contain numbers, valuables, and +, −, ×, ÷ signs but not = |
| **equation** | $4x = 8$ | a sentence with an equal sign |
| **inequality** | $7x < 30$ or $x \neq 6$ | a sentence with one of the following signs: $\neq$, $<$, $>$, $\leq$, $\geq$, or $\approx$ |
| **base** | $6^3$<br>(The base is 6.) | the number used as a factor |
| **exponent** | $6^3$<br>(The exponent is 3.) | the number of times the base is multiplied by itself |



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**Example 1:** Look at the following expression and identify each part.

$3x^4 + 5x + 6$

There are 3 *terms*: $3x^4$, $5x$, and 6.

There are 3 *coefficients*: 3, 5, and 6, and 3 is the *leading coefficient*.

$x$ is a *variable*, and 4 is an *exponent*. The *degree* is 4.

**Identify the parts of the expressions below. (DOK 1)**

$5n^2 - 3$

1. How many terms are there in the expression?

_____

2. What does $n$ represent in the expression?

---

$3b^2 + 2b^4 - 3b + 4$

3. What does $b$ represent in the expression?

4. What is the leading coefficient?

---

$(x + a)(x - b)$

5. What is the degree?

6. How many terms are in the expression?

---

$3x^2 + 5xy^2 + y^4$

7. What is the leading coefficient?

8. What is the degree?

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## 3.2 Translating Expressions (DOK 1)

The biggest challenge to solving word problems is figuring out whether to add, subtract, multiply, or divide. Below is a list of key words and their meanings. This list does not include every situation you might see, but it includes the most common examples.

| Words Indicating Addition | Example | Add |
|---|---|---|
| and | 6 **and** 8 | 6 + 8 |
| increased | The original price of \$15 **increased** by \$5. | 15 + 5 |
| more/more than | Josh has 10 points. Will has 5 **more than** Josh. | 10 + 5 |
| plus | 8 baseballs **plus** 4 baseballs | 8 + 4 |
| sum | the **sum** of 3 and 5 | 3 + 5 |
| total | the **total** of 10, 14, and 15 | 10 + 14 + 15 |

| Words Indicating Subtraction | Example | Subtract |
|---|---|---|
| decreased | \$16 **decreased** by \$5 | 16 − 5 |
| difference | the **difference** between 18 and 6 | 18 − 6 |
| less | 14 days **less** 5 | 14 − 5 |
| less than | Jose completed 2 laps **less than** Mike's 9. | *9 − 2 |
| left | Ray sold 15 out of 35 tickets. How many did he have **left**? | *35 − 15 |
| lower than | This month's rainfall is 2 inches **lower than** last month's rainfall of 8 inches. | *8 − 2 |
| minus | 15 **minus** 6 | 15 − 6 |

*In subtraction word problems, you cannot always subtract the numbers in the order that they appear in the problem. Read the problem carefully to determine if the first number should be subtracted from the last.

| Words Indicating Multiplication | Example | Multiply |
|---|---|---|
| double | Her \$1,000 profit **doubled** in a month. | 1,000 × 2 |
| product | the **product** of 4 and 8 | 4 × 8 |
| times | Li scored 3 **times** as many points as Ted who only scored 4. | 3 × 4 |
| triple | The bacteria **tripled** its original colony of 10,000 in just one day. | 3 × 10,000 |
| twice | Ron has 6 CDs. Tom has **twice** as many. | 2 × 6 |

| Words Indicating Division | Example | Divide |
|---|---|---|
| divide into, by, or among | The group of 70 **divided into** 10 teams | $70 \div 10$ or $\frac{70}{10}$ |
| quotient | the **quotient** of 30 and 6 | $30 \div 6$ or $\frac{30}{6}$ |

| Words Indicating Equal Sign | Example | Equation |
|---|---|---|
| is, are | Jake's age, $x$, minus 4 **is** 22. | $x - 4 = 22$ |
| equal(s) | 9 stamps + 3 stamps **equals** 12 stamps. | 9 + 3 = 12 |


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**Match the phrase with the correct algebraic expression below. The answers will be used more than once. (DOK 1)**

A. $y - 2$ B. $2y$ C. $y + 2$ D. $\frac{y}{2}$ E. $2 - y$

1. 2 less than $y$
2. 2 more than $y$
3. $y$ divided by 2
4. 2 less $y$
5. the product of 2 and $y$
6. $y$ decreased by 2

**Now practice writing parts of algebraic expressions from the following word problems. (DOK 1)**

7. 3 less than $x$
8. $y$ divided among 10
9. 5 times $k$
10. the total of $z$ and 12
11. 18 divided by a number, $x$
12. 3 more than $p$

## 3.3 Substituting Numbers for Variables (DOK 1)

These problems may look difficult at first glance, but they are very easy. Simply replace the variable with the number the variable is equal to, and solve the problems.

**Example 1:** In the following problems, substitute 10 for $a$.

| Problem | Calculation | Solution |
|---|---|---|
| 1. $a + 1$ | Simply replace the $a$ with 10. $10 + 1$ | 11 |
| 2. $17 - a$ | $17 - 10$ | 7 |
| 3. $9a$ | This means multiply. $9 \times 10$ | 90 |
| 4. $\frac{30}{a}$ | This means divide. $30 \div 10$ | 3 |
| 5. $a^3$ | $10 \times 10 \times 10$ | 1000 |
| 6. $5a + 6$ | $(5 \times 10) + 6$ | 56 |

**Note: Be sure to do all multiplying and dividing before adding and subtracting.**

**Example 2:** In the following problems, let $x = 2$, $y = 4$, and $z = 5$.

| Problem | Calculation | Solution |
|---|---|---|
| 1. $5xy + z$ | $5 \times 2 \times 4 + 5$ | 45 |
| 2. $xz^2 + 5$ | $2 \times 5^2 + 5 = 2 \times 25 + 5$ | 55 |
| 3. $\frac{yz}{x}$ | $(4 \times 5) \div 2 = 20 \div 2$ | 10 |

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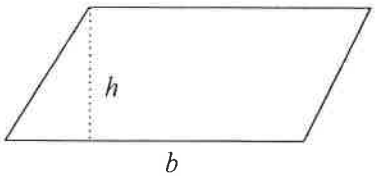
You can also substitute numbers for variables in a formula.

**Example 3:** Area of a parallelogram: $A = b \times h$
Find the area of the parallelogram if $b = 20$ cm and $h = 10$ cm.




**Step 1:** Solve the formula with the numbers given in place of the letter in the formula.
$A = 20 \times 10 = 200$. Therefore, $A = 200\ cm^2$.

**In the following problems, $t = 7$. Solve the problems. (DOK 1)**

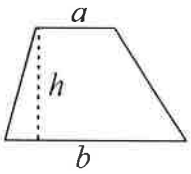
1. $3t - 5 =$
2. $9t \div 3 =$
3. $4t + 5t =$
4. $\dfrac{(t^2 - 7)}{6} =$

**In the following problems $a = 4$, $b = -2$, $c = 5$, and $d = 10$. Solve the problems. (DOK 1)**

5. $4a + 2c =$
6. $abd =$
7. $5c - ad =$
8. $\dfrac{6b}{a} =$

**Solve the following problems using the formulas given. (DOK 1)**

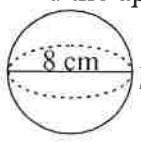
9. Find the area of the trapezoid




$A = \frac{1}{2}h(a + b)$
$a = 11$ in
$b = 23$ in
$h = 18$ in

10. The formula for changing from degrees Fahrenheit to degrees Celsius is: $C = \dfrac{5(F - 32)}{9}$
If it is 68 °F outside, how many degrees Celsius is it?

11. Find the approximate volume.




$V = \frac{4}{3}\pi r^3$ $\pi \approx 3.14$

12. Louise has a cone-shaped mold to make candles. The diameter of the base is 10 cm, and it is 13 cm tall. About how many cubic centimeters of liquid wax will it hold?

$\pi \approx 3.14$ $V = \frac{1}{3}\pi r^2 h$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## Chapter 3 Review

**Solve the following problems using $x = 2$. (DOK 1)**

1. $\frac{6x}{4} =$
2. $12 - 3x =$
3. $-5x + 4 =$
4. $\frac{x^3+8}{2} =$

**Solve the following problems. Let $w = -1$, $y = 3$, and $z = 5$. (DOK 1)**

5. $wyz + 2 =$
6. $\frac{3z+5}{wz} =$
7. $z - 2w =$
8. $\frac{6w}{y} + \frac{z}{w} =$

**Identify the parts of each expression as required. (DOK 1)**

$4x^2 - 3x^5 + x^3 - 8x$

9. What is the degree?

10. How many terms are in the expression?

11. What does $x$ represent?

12. Omar is a salesman. He earns an hourly wage of $8.00 per hour, plus he receives a commission of 7% on the sales he makes. Write an equation which would be used to determine his weekly salary, $w$, where $x$ is the number of hours worked, and $y$ is the amount of sales for the week.

**Match each set of algebraic expressions with the correct descriptive phrase. (DOK 1)**

13. $8(y - 2)$
14. $\frac{y-2}{8}$
15. $8y - 2$
16. $\frac{y}{8} - 2$

A. two less than the product of $y$ and 8

B. the difference of $y$ and 2 divided by 8

C. two less than one-eighth of $y$

D. eight times the difference of $y$ and 2

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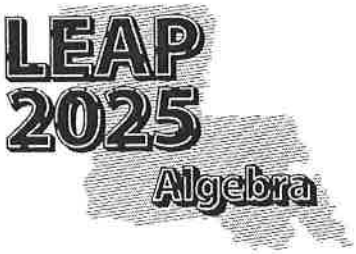
# Chapter 4
# Solving Word Problems




Standards covered: N.Q.A.1, N.Q.A.2, N.Q.A.3, A.CED.A.1, A.CED.A.3

## 4.1 Algebra Word Problems (DOK 2)

An equation states that two mathematical expressions are equal. In working with word problems, the words that mean equal are **equals, is, was, is equal to, amounts to**, and other expressions with the same meaning. To translate a word problem into an algebraic equation, use a variable to represent the unknown or unknowns you are looking for.

In the following example, let $n$ be the number you are looking for.

**Example 1:** Four more than twice a number is two less than three times the number.

**Step 1:** Translation: $4 + 2n = 3n - 2$

**Step 2:** Now solve:

$$\begin{array}{rcl} -2n & & -2n \\ \hline 4 & = & n-2 \\ +2 & & +2 \\ \hline 6 & = & n \end{array}$$

**Answer:** The number is 6.

Substitute the number back into the original equation to check.

**Translate the following word problems into equations and solve. (DOK 2)**

1. Four less than twice a number is ten. Find the number.

2. Seven times a number plus the number is 24. What is the number?

3. Negative 14 is equal to ten minus the product of six and a number. What is the number?

4. Fluffy, Spot, and Shampy have a combined age in dog years of 91. Spot is 14 years younger than Fluffy. Shampy is 6 years older than Fluffy. What is Fluffy's age, $f$, in dog years?

5. Abby is a server at a local restaurant. She makes an hourly wage of $3.50, plus she receives tips. On Monday, she works 6 hours and receives tip money, $t$. Write an equation showing what Abby makes on Monday, $y$.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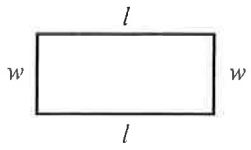
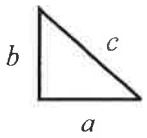
## 4.2 Word Problems with Formulas (DOK 2)

You can solve word problems with formulas the same way as regular word problems. Simply take what the world problem tells you about the variables and plug them into the formula. Then, solve the formula as you normally would.




perimeter = $2l + 2w$




perimeter = $a + b + c$

**Example 1:** The perimeter of a geometric figure is the distance around the outside of the figure. The perimeter of a rectangle is 44 feet. The length of the rectangle is 6 feet more than the width. What is the measure of the width?

**Step 1:** Let the variable be the length of the unknown side.
width = $w$ length = $6 + w$

**Step 2:** Use the equation for the perimeter of a rectangle as follows:
$2l + 2w$ = perimeter of a rectangle
$2(w + 6) + 2w = 44$

**Step 3:** Solve for w.

| | |
|---|---|
| $2(w + 6) + 2w = 44$ | |
| $2w + 12 + 2w = 44$ | Distribute. |
| $4w + 12 = 44$ | Combine like terms. |
| $4w = 32$ | Subtract 12 from both sides of the equation. |
| $w = 8$ | Divide both sides of the equation by 4. |

**Answer:** width = 8 feet

Always plug your answer back into the original equation to check your answer.

**Solve the following word problems. (DOK 2)**

1. The length of a rectangle is 3 more than twice the width. The perimeter is 36. What is the length?

2. The perimeter of a triangle is 18 feet. The second side is two feet longer than the first. The third side is two feet longer than the second. What are the lengths of the sides?

3. In an isosceles triangle, two sides are equal. The third side is two less than twice the length of the sum of the two sides. The perimeter is 40. What are the lengths of the three sides?

4. The sum of the measures of the angles of a triangle is 180°. The second angle of a triangle is twice the measure of the first angle. The third angle is 20 more than 5 times the first. What are the measures of the three angles?

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## 4.3 Inequality Word Problems (DOK 1)

Inequality word problems involve staying under a limit or having a minimum goal one must meet.

**Example 1:** A contestant on a popular game show must earn a minimum of 800 points by answering a series of questions. The contestant will answer questions from four categories. Her results for the first three categories are as follows: 160 points, 200 points, and 240 points. Write an inequality which describes how many points, ($p$), the contestant will need on the last category in order to win.

**Step 1:** Write an inequality to describe the situation. $160 + 200 + 240 + p \geq 600$

**Step 2:** Solve the inequality.

$$\begin{array}{rl} 600 + p \geq & 800 \\ -600 \qquad & -600 \\ \hline p \geq & 200 \end{array}$$

The contestant must get 200 or more points on the last category.

**Solve each of the following problems using inequalities. (DOK 1)**

1. A high school band program will receive $2,000.00 for selling $12,000.00 worth of coupon books. Six band classes participate in the sales drive. Classes 1–5 collect the following amounts of money: $2,400, $2,800, $1,500, $2,320, and $2,550. Write an inequality which describes the amount of money the sixth class must collect so that the band will receive $2,000.

2. A small elevator has a maximum capacity of 1,200 pounds before the cable holding it in place snaps. Six people get on the elevator. Five of their weights follow: 120, 240, 150, 215, and 170. Write an inequality which describes the amount the sixth person can weigh without snapping the cable.

3. A small high school class of 9 students were told they would receive a pizza party if their class average was 90% or higher on the next exam. Students 1–8 scored the following on the exam: 84, 95, 99, 87, 92, 93, 100, and 98. Write an inequality which describes the score the ninth student must make for the class to qualify for the pizza party.


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## 4.4 Rates (DOK 1)

Rates are often discussed in terms of miles per hour, but a rate can be any measured quantity divided by another measurement such as feet per second, kilometers per minute, mass per unit volume, etc. A rate can be how fast something is done. For example, a bricklayer may lay 80 bricks per hour. Rates can also be used to find measurements such as density. For example, 35 grams of salt in 1 liter of water gives the mixture a density of 35 grams/liter.

Here is the formula for rate of speed: $\text{Rate} = \frac{\text{distance}}{\text{time}}$ or $R = \frac{d}{t}$.

**Example 1:** Laurie traveled with her family 312 miles to the Grand Canyon in 6 hours. What was her average rate of speed?

Divide the number of miles by the number of hours. $\frac{312 \text{ miles}}{6 \text{ hours}} = \frac{52 \text{ miles}}{1 \text{ hour}}$

**Answer:** Laurie's average rate of speed was 52 miles per hour or 52 miles/hour.

**Example 2:** Nathan entered his snail in a race. His snail went 18 feet in 6 minutes. How fast did his snail move?

In this problem, the units given are feet and minutes, so the rate will be feet per minute (or feet/minute).

You need to find out how far the snail went in one minute.

$\text{Rate} = \frac{\text{distance}}{\text{time}}$ $\qquad$ $R = \frac{18 \text{ feet}}{6 \text{ minutes}} = \frac{3 \text{ feet}}{1 \text{ minute}}$

**Answer:** Nathan's snail went an average of 3 feet per minute or 3 ft/min.

**Find the average rate for each of the following problems. (DOK 1)**

1. A commercial jet traveled 1,572 miles in 3 hours. What was its average speed?

2. The Declaration of Independence is approximately 1,458 words including signatures, which Tewanda read in 9 minutes. How fast did she read the document?

3. Spectators at the Super Circus were amazed to watch a cannon shoot a clown 212 feet into a net in 4 seconds. How many feet per second did the clown travel?

4. Marcus Page, star receiver for the Big Bulls, was awarded a 5-year contract for 105 million dollars. How much will his annual rate of pay be if he is paid the same amount each year?

5. The new McDonald's in Moscow serves 11,208 customers during a 24-hour period. What is the average number of customers served per hour?

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## 4.5 Dimensional Analysis (DOK 2)

A **dimension** is a property that can be measured such as length, time, mass, or temperature; or it is calculated by multiplying or dividing other dimensions. Some examples include length/time (velocity), length$^3$ (volume), or mass/length$^3$ (density).

A measured quantity can be expressed in any appropriate dimension. The equivalence between two expressions of a given quantity may be written as a ratio:

$$\frac{16 \text{ ounces}}{1 \text{ pound}} \text{ or } \frac{2000 \text{ pounds}}{1 \text{ ton}}$$

Ratios of equivalent values expressed in different units like these are known as **conversion factors**. To convert given quantities in one set of units to their equivalent values in another set of units, we set up **dimensional equations**. Writing these with vertical and horizontal bars and carrying along units often helps avoid mistakes in these types of equations.

**Example 1:** How many inches are in four yards?

**Step 1:** First, you need to develop our conversion factors. You have a value in yards and you need the value in inches. There are 3 feet in 1 yard and 12 inches in 1 foot, so you can set up a dimensional equation using these conversion factors. Cross out the units top and bottom to make sure that you are left with the units that you want. Then, multiply the numbers across the top and divide by the numbers on the bottom to get the answer.

$$\frac{4 \text{ yards} \mid 3 \text{ feet} \mid 12 \text{ inches}}{\quad \mid 1 \text{ yard} \mid 1 \text{ foot}} = \frac{(4) \times (3) \times (12)}{(1) \times (1)} = 144 \text{ inches}$$

You are often asked to convert the units of area or volume, which are square or cubic terms. It is important to note that the numerical coefficient must also be squared or cubed when converting units that are squared or cubed.

**Example 2:** The volume of a barrel is 10 ft$^3$. How many in$^3$ of water will it hold?

$$\frac{10 \text{ ft}^3 \mid (12)^3 \text{ in}^3}{\quad \mid 1 \text{ ft}^3} = \frac{(10) \times (12) \times (12) \times (12)}{(1)} = 17{,}280 \text{ in}^3$$

**Example 3:** How many kilograms per cubic meter (kg/m$^3$) are there in 3 grams per cubic centimeter (g/cm$^3$)?

$$\frac{3 \text{ grams} \mid 1 \text{ kg} \mid (100)^3 \text{ cm}^3}{\text{cm}^3 \mid 1000 \text{ grams} \mid 1 \text{ m}^3} = 3000 \frac{\text{kg}}{\text{m}^3}$$

This is an example of converting units of **density**, which is a measure in mass per unit volume.

Another useful scientific term which often needs converting is **velocity**, measured in length per unit time.

**Example 4:** What is the highway speed limit of 65 miles per hour in feet per second?

$$\frac{65 \text{ miles} \mid 5{,}280 \text{ feet} \mid 1 \text{ hour} \mid 1 \text{ minute}}{1 \text{ hour} \mid 1 \text{ mile} \mid 60 \text{ minutes} \mid 60 \text{ seconds}} = 95.3 \frac{\text{feet}}{\text{second}}$$



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**Solve these problems. (DOK 2)**

1. Jared can work 54 math problems in one hour. How many problems can he work in 10 minutes?

2. Leah rides 22 feet per second on her bicycle. How many miles per hour does she ride?

3. A jar of honey has a density of 14 kg per $m^3$. What is its density in $g/mm^3$?

4. Deandre throws his fastball 99 miles per hour. If he starts 60 feet, 6 inches from home plate, how many seconds does it take the ball to get to the plate?

## 4.6 Accuracy, Precision, and Error (DOK 2)

When you perform measurements or solve word problems, you will have answers in units. These units must not only be appropriate to the situation, but they must be precise as well as accurate to ensure the quality of your answer. Think about this: would you measure the amount of water in liters or tons? Units must be defined appropriately. Now, consider this: does measuring from the end mark of a ruler rather than the end of the ruler make a difference? What is the difference between precision and accuracy? Many people use these terms interchangeably, but they are not the same.

**Accuracy** refers to how "correct" a measurement is. The more "correct" a measurement is compared to the actual value, the more accurate it is.

**Precision** refers to the level of detail in a number or estimate. A precise number has many significant digits. The smaller the scale, the more precise the measurement. For example, a measurement made to the nearest eighth of an inch is more precise than a measurement that is rounded to the nearest inch.

**Errors** in precision and accuracy result in more work.

**Note:** An answer may be precise *without* being accurate.

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## Accuracy

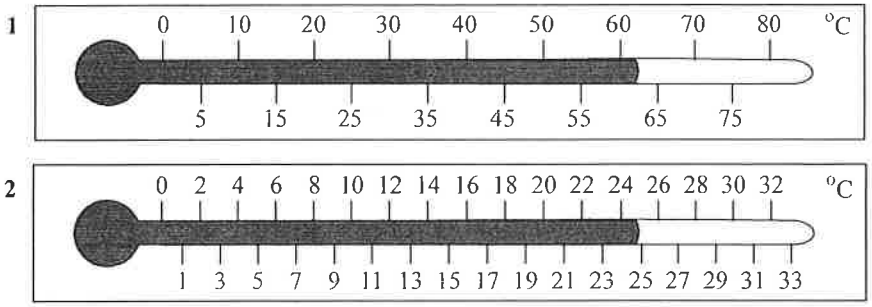
**Example 1:** What is an accurate estimate of a yard stick?

An accurate estimate of a yard stick is 3 feet, but a more precise estimate of a yard stick is 3.1 feet.

Precision does not always have to be accurate. A more precise measurement is not necessarily more accurate. **It can be more precise, but wrong.**

## Precision

**Example 1:** Look at the 2 thermometers below. Which one would give you a more precise temperature reading?




**Answer:** The second thermometer would give a more precise temperature reading because it measures in 1-degree increments. The first thermometer only measures in 5-degree increments.

**Example 2:** Which of the following measurements is most precise?

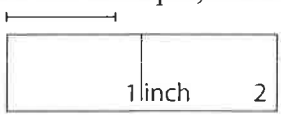
**A** $3\frac{1}{4}$ in **B** $3\frac{1}{16}$ in **C** 3 in **D** $3\frac{1}{32}$ in

**Answer:** Choice D is the most precise measurement because it is given in the smallest scale. This choice indicates that the ruler used to measure was marked in thirty-seconds of an inch. Remember, the smallest fraction indicates the greatest precision. Choice A indicates that the instrument used measured to the nearest one-fourth of an inch, choice B to the nearest sixteenth, and choice C to the nearest inch.

**Note:** What if choice A were written as $3\frac{8}{32}$ ? Then the precision of choice A and the precision of choice D would be the same. They would both show that the ruler is measured to the nearest thirty-second of an inch. **When showing precision in a measurement, you do not reduce the fraction**.

## Error

Since no instrument can measure with perfect precision, all measurements are subject to a certain amount of error. This error does not refer to human error but to error in rounding. The more precise the scale, the smaller the possible error. For example, measure the line segment below.





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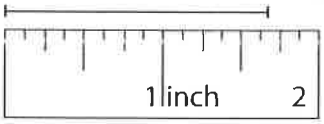
According to this ruler, the line segment is close to 1 inch. You have to round to the nearest inch to find the approximate value. The result of rounding the measurement is called error of measurement because the true measurement is really a little less than the stated measurement of 1 inch.

So, what is the maximum error in a measurement? Look at the line segment above. Since the line segment is more than half way between 0 and 1, you round to 1. Therefore, the error in measuring line segments with this particular ruler will be half of the distance between 0 and 1. The maximum error is half an inch.

**Example 1:** What is the measurement of the line segment below? What is the precision of the ruler? What is the maximum error of a line segment measured with this ruler?




**Answer:** The line segment is $1\frac{5}{8}$ inches. The precision of the ruler is to the nearest one-eighth of an inch. The maximum error of this ruler is one-half of one-eighth of an inch which is $\frac{1}{16}$ of an inch. **The maximum error is one-half the unit of measure.**

**Example 2:** A map gives a scale in increments of ten miles. What is the maximum error when measuring between two cities on the map?

**Answer:** Since the scale is ten miles, the maximum error would be 5 miles.

**Circle the most precise measurement. (DOK 2)**

1. A. $6\frac{2}{8}$ c  B. $4\frac{1}{4}$ c  C. $6\frac{4}{16}$ c  D. $7\frac{8}{32}$ c  E. $4\frac{5}{16}$ c
2. A. 1 oz  B. 0.0150 oz  C. 0.015 oz  D. 1.015 oz  E. 1.5 oz

**Answer the following questions. (DOK 2)**

3. Which would give you a more precise measurement: a graduated cylinder marked in mL or a graduated cylinder marked in dL?

4. Which would give you a more precise measurement: a pressure gauge that measures in pounds or a pressure gauge that measures in tenths of pounds?

**Find the maximum error for the following problems. (DOK 2)**

5. a ruler marked in sixteenths of an inch

6. a map scale marked in 50-mile increments

7. a scale marked in tenths of a pound

8. exit sign numbers designated by the nearest mile

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## Chapter 4 Review

**Solve each of the following problems. (DOK 1, 2)**

1. The band members sold tickets to their concert performance. Some were \$3 tickets, and some were \$8 tickets. There were 10 more than twice as many \$8 tickets sold as \$3 tickets. The total sales were \$1,448. How many tickets of each price were sold?

2. One number is 8 more than the other number. Twice the smaller number is 7 more than the larger number. What are the numbers?

3. The perimeter of a triangle is 48 inches. The second side is four inches longer than the first side. The third side is one inch longer than the second. Find the length of each side.

4. Joe, Craig, and Dylan have a combined weight of 326 pounds. Craig weighs 40 pounds more than Joe. Dylan weighs 12 pounds more than Craig. How many pounds does Craig weigh?

5. Lena and Jodie are sisters, and together they have 56 bottles of nail polish. Lena bought 4 more than half the bottles. How many did Jodie buy?

6. Jim takes great pride in decorating his float for the homecoming parade for his high school. With the \$5,000 he has to spend, Jim buys 5,000 carnations at \$0.30 each, 4,000 tulips at \$0.60 each, and 300 irises at \$0.25 each. Write an inequality which describes how many roses, $r$, Jim can buy if roses cost \$0.80 each.

7. Mr. Chan wants to sell some or all of his shares of stock in a company. He purchased the 90 shares for \$0.50 each last month, and the shares are now worth \$3.80 each. Write an inequality which describes how much profit, $p$, Mr. Chan can make by selling his shares.

8. Lumber is measured with the following formula:

   Number of board feet $= \dfrac{LWT}{12}$

   $L$ = Length of the board in feet

   $W$ = Width of the board in feet

   $T$ = Thickness of the board in feet

   Find the number of board feet if $L = 12$ feet, $W = 6$ feet, and $T = \frac{1}{4}$ feet.


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9. To convert from degrees Celsius to degrees Fahrenheit, use the following formula:

$$F = \frac{9C}{5} + 32$$

If it is 15 °C outside, what is the temperature in degrees Fahrenheit?

10. The Jones family traveled 300 miles in 5 hours. What was their average speed?

11. Last year, Rikki sang 960 songs with her rock band. How many songs did she sing per month?

**Using dimensional analysis, make the following conversions. (DOK 1)**

12. 15 yards to feet

13. 3 pounds to ounces

**Choose the most precise measurement. (DOK 2)**

| | | | |
|---|---|---|---|
| 14. | **A** 0.5 | **B** 0.05 | **C** 0.005 |
| 15. | **A** 4 | **B** 4.7 | **C** 4.81 |
| 16. | **A** $5\frac{1}{2}$ | **B** $5\frac{1}{4}$ | **C** $5\frac{1}{8}$ |

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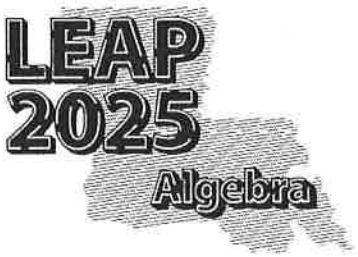
# Chapter 5
# Polynomials




Standard covered: A.APR.A.1

## 5.1 Identifying Polynomials

**Polynomials** are algebraic expressions which include more than one term, or **monomial. Binomials** contain two terms, and **trinomials** contain three terms. Expressions with more than three terms are simply called **polynomials**. **Terms** are separated by plus and minus signs.

**Examples:**

| Monomials | Binomials | Trinomials | Polynomials |
|---|---|---|---|
| 4 | $4t + 9$ | $x^2 + 2x + 3$ | $x^3 - 3x^2 + 3x - 9$ |
| $3x$ | $9 - 7g$ | $5x^2 - 6x - 1$ | $p^4 + 2p^3 + p^2 - 5 + p$ |
| $4g^2$ | $5x^2 + 7x$ | $y^4 + 15y^2 + 100$ | |

**For each of the following questions, determine the number of terms in the expression and label as monomial, binomial, trinomial, or polynomial. (DOK 1)**

1. $-9y^4$
2. $7z^3 + 2z^2 - 3z + 4$
3. $10 + 3m$
4. $12j^2 - 4j + 6$
5. $5p^5 - 4p^4 + 3p^3 - 2p^2 + p - 6$
6. 0

## 5.2 Adding Polynomials (DOK 2)

When adding polynomials, make sure the exponents and variables are the same on the terms you are combining. Write the terms with like exponents in columns under each other. Add each column separately. Fill in the blank spots with zeros if it helps you keep the columns straight. **You never carry to the next column when adding polynomials.**

**Example 1:** Add $3x^2 + 14$ and $5x^2 + 2x$

$$\begin{array}{r} 3x^2 + 0x + 14 \\ (+)\,5x^2 + 2x + 0 \\ \hline 8x^2 + 2x + 14 \end{array}$$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**Add the following polynomials. (DOK 2)**

1. $(5y^2 + 4y - 6) + (2y^2 - 5y + 8)$
2. $(3s^3 + s^2 - 2) + (-2s^3 + 4)$
3. $(-v^2 + 7v - 8) + (4v^3 - 6v + 4)$
4. $(3p^4 + 2p^2 - 1) + (-5p^2 - p + 8)$
5. $(y^2 - 11y + 10) + (-13y^2 + 5y - 4)$
6. $(25z^2 + 13z + 8) + (z^2 - 2z - 10)$

## 5.3 Subtracting Polynomials (DOK 2)

When you subtract polynomials, it is important to remember to change all the signs in the subtracted polynomial (the subtrahend) and then add.

**Example 1:** $(4y^2 + 8y + 9) - (2y^2 + 6y \quad 4)$

**Step 1:** Write the subtraction problem in vertical form. Make sure you line up the terms with like exponents under each other.

$$\begin{array}{r} 4y^2 + 8y + 9 \\ (-)(2y^2 + 6y - 4) \\ \hline \end{array}$$

**Step 2:** Change the subtraction sign to addition and all the signs of the subtracted polynomial to the opposite sign.

$$\begin{array}{r} 4y^2 + 8y + 9 \\ (+)-2y^2 - 6y + 4 \\ \hline 2y^2 + 2y + 13 \end{array}$$

**Subtract the following polynomials. (DOK 2)**

1. $(8y - 4) - (4y + 3)$
2. $(11t^3 - 4t^2 + 3) - (-t^3 + 4t^2 - 5)$
3. $(-3w^2 + 9w - 5) - (-5w^2 - 5)$
4. $(13y^3 + 2y^2 - 8y) - (2y^3 + 4y^2 - 7y)$
5. $(7g + 3) - (g^2 + 4g - 5)$
6. $(4b^3 + 3b^2 + 5) - (7b^3 - 8)$

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## 5.4 Simplifying Polynomials (DOK 2)

In the following problem, you must distribute the variables outside the parentheses and then combine the like terms to simplify the expressions.

**Example 1:** $8x\,(2x^2 - 5x + 7) - 3x\,(4x^2 + 3x - 8)$

**Step 1:** Distribute to remove the first set of parentheses.

$8x\,(2x^2 - 5x + 7) = 16x^3 - 40x^2 + 56x$

**Step 2:** Distribute to remove the second set of parentheses. Remember to keep the (–) sign with $-3x$.

$-3x\,(4x^2 + 3x - 8) = -12x^3 - 9x^2 + 24x$

**Step 3:** Write each polynomial in columns, making sure the terms with the same variable and exponent are under each other. Combine like terms.

$$\begin{array}{r} 16x^3 - 40x^2 + 56x \\ (+)\ -12x^3 - 9x^2 + 24x \\ \hline 4x^3 - 49x^2 + 80x \end{array}$$

**Remove the parentheses and simplify the following problems. (DOK 2)**

1. $4t\,(t + 7) + 5t\,(2t^2 - 4t + 1)$
2. $-5y\,(3y^2 - 5y + 3) - 6y\,(y^2 - 4y - 4)$
3. $-3\,(3x^2 + 4x) + 5x\,(x^2 + 3x + 2)$
4. $8d^2\,(3d + 4) - 7d\,(3d^2 + 4d + 5)$
5. $4c^2\,(-6c^2 - 3c + 2) - 7c\,(5c^3 + 2c)$
6. $-8w\,(-w + 1) - 4w\,(3w - 5)$

## 5.5 Evaluating Polynomials for Given Values (DOK 2)

Now that you know how to identify polynomials, you will learn how to evaluate them for a given value.

**Example 1:** Evaluate $y^4 + 3y + 4$ when $y = 2$.

**Step 1:** Substitute 2 in for y.

$(2)^4 + 3(2) + 4$

**Step 2:** Perform operations.

$(2)^4 + 3(2) + 4 = 16 + 6 + 4$

**Step 3:** Simplify by adding terms.

$16 + 6 + 4 = 26$



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**Evaluate the polynomials for the given values. (DOK 2)**

1. $3x^4 - x + 4; x = 3$
2. $3x^4 - 25x^3 + 27; x = 2$
3. $20s^4 + 24s^3 + 4s; s = 1$
4. $6m^4 - 4m + 20; m = 4$
5. $-7p^4 - p + 9; p = -2$
6. $6a^7 - a^3 + a; a = -1$

## 5.6 Multiplying Two Binomials (DOK 2)

When you multiply two binomials such as $(x + 6)(x - 7)$, you must multiply each term in the first binomial by each term in the second binomial. If you can remember the word **FOIL**, it can help you keep order when you multiply. The "***F***" stands for **first**, "***O***" stands for **outside**, "***I***" stands for **inside**, and "***L***" stands for **last**.

| **F** | **O** | **I** | **L** |
|---|---|---|---|
| **FIRST** | **OUTSIDE** | **INSIDE** | **LAST** |
| Multiply the first terms in each binomial | Next, multiply the outside terms. | Then, multiply the inside terms. | Last, multiply the last terms. |
| $(x + 6)(x - 7)$ | $(x + 6)(x - 7)$ | $(x + 6)(x - 7)$ | $(x + 6)(x - 7)$ |
| $x \cdot x = x^2$ | $x \cdot (-7) = -7x$ | $6 \cdot x = 6x$ | $6 \cdot (-7) = -42$ |
| $x^2$ | $-7x$ | $6x$ | $-42$ |

Now, just combine like terms and write your answer.

**Solution:** $(x + 6)(x - 7) = x^2 - x - 42$

**Note:** Be sure to write your answers in descending order. That means that the term with the highest-number exponent comes first in a polynomial. The next highest exponent is second and so on.

**Multiply the following binomials. Show your work. (DOK 2)**

1. $(4x + 5)(x + 20)$
2. $(5b - 3)(-5 + 3b)$
3. $(8k - 7)(-5k - 3)$
4. $(5y - 20)(5y + 20)$

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5. $(a + 6)(3a + 7)$

6. $(6t - 20)(5t - 5)$

7. $(2x + 1)(2x + 1)$

8. A school changed their playground from a square shape into a rectangle to accommodate the remodel of the gymnasium. They added 50 feet to one side of the playground and decreased the other side by 20 feet.

   (A) Write an expression that could be used to express the area of the new playground.

   (B) Evaluate the expression assuming the original playground has the given side lengths: 50 ft, 100 ft, and 150 ft.

   (C) Do you think there will ever be a time when the new playground will not have a larger area than the old playground? Explain your reasoning.

## Chapter 5 Review

**Simplify.**

1. $-6z(z + 3)$

2. $(4b)(5b)$

3. $7x - 9x$

4. $(5p - 4) - (3p + 2)$

5. $3(2g + 3)$

6. $4y(4y - 9y + 2)$

7. $8x^2 + 12x^2$

8. $15p^3 - 11p^3$

9. $2b(b - 4) - (b^2 + 2b + 1)$

10. $(5d + 20)(4d + 8)$



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**Evaluate the polynomials for the given values.**

11. $5x^3 - 2x + 6; x = -2$

12. $2x^2 - 5x + 12; x = 5$

13. $14s^3 - 7s^2 + 16s; s = -1$

14. $-8w^3 + 15w + 19; w = 7$

**Carefully read the problem below and answer the questions that follow. (DOK 3)**

15. $8x^2 - 7$, $3x^2 + 4x$, and $5x^2 - 6$ represent three sides of a triangle.

A) Write an expression that represents the perimeter of this triangle.

B) What would the perimeter of the triangle be if $x = 3$?

C) What would each side length of the triangle be if $x = 3$?

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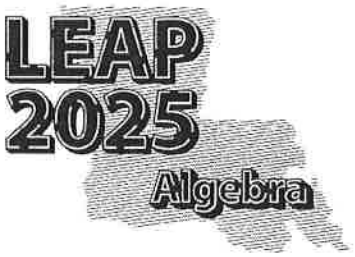
# Chapter 6
# Equations and Inequalities




Standards covered: A.CED.A.1, A.CED.A.3, A.CED.A.4, A.REI.A.1, A.REI.A.3

## 6.1 Two-Step Algebra Problems (DOK 2)

Sometimes it takes more than one step to solve an equation. What is done to one side of the equation must be done to the other side. In the following two-step algebra problems, **addition** and **subtraction** are performed first and then **multiplication** and **division**.

**Example 1:** $-4x + 7 = 31$

**Step 1:** Subtract 7 from both sides.

$$\begin{array}{rrr} -4x & + 7 & = 31 \\ & -7 & -7 \\ \hline -4x & & = 24 \end{array}$$

**Step 2:** Divide both sides by $-4$.

$$\frac{\cancel{-4}x}{\cancel{-4}} = \frac{24}{-4} \quad \text{so} \quad x = \frac{24}{-4} = -6$$

An algebra problem may contain a fraction. Study the following example to understand how to solve algebra problems that contain a fraction. Remember, what you do to one side of your equation must be done to the other side.

**Example 2:** $\frac{x}{2} + 4 = 3$

**Step 1:**

$$\begin{array}{rrr} \frac{x}{2} & + 4 & = 3 \\ & -4 & -4 \\ \hline \frac{x}{2} & & = -1 \end{array}$$

Subtract 4 from both sides.

**Step 2:** $\frac{x}{2} = -1$ Multiply both sides by 2 to eliminate the fraction.

$$\frac{x}{\cancel{2}} \times \cancel{2} = -1 \times 2 \quad \text{so} \quad x = -2$$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Study the following example to understand how to solve algebra problems that contain a different type of fraction.

**Example 3:** $\frac{x+2}{4} = 3$ In this example, "$x + 2$" is divided by 4, and not just the $x$ or the 2.

**Step 1:** $\frac{x+2}{\cancel{4}} \times \cancel{4} = 3 \times 4$ First, multiply both sides by 4 to eliminate the fraction.

$x + 2 = 12$

**Step 2:**
$$\begin{array}{r} x + 2 = 12 \\ -2 \quad -2 \\ \hline x = 10 \end{array}$$
Next, subtract 2 from both sides.

**Solve the two-step algebra problems below. Show your work. (DOK 2)**

1. $\frac{w}{5} - 3 = 6$
3. $6y - 14 = 34$
5. $15 + \frac{z}{14} = 13$
2. $-6b - 8 = 10$
4. $21t + 17 = 80$
6. $-5 + 36w = 175$

**Solve the following problems. Show your work. (DOK 2)**

7. $\frac{w-10}{-8} = -4$
8. $\frac{13+h}{2} = 12$
9. $\frac{w+2}{15} = -1$

## 6.2 Equations with Like Terms (DOK 2)

Some equations can have like terms in them. Like terms share the same variables and exponents, but the coefficients do not have to match. Like terms can be easily added together to make one term.

**Example 1:** $9b$ and $4b$ are like terms. These terms can be combined.
$9b + 4b = 13b$

$12b$ and $b^2$ are not like terms. One has an exponent and the other does not. These terms cannot be combined.

When an equation has two or more like terms on the same side of the equation, combining like terms is the first step in solving the equation.

**Example 2:** $7x + 2x - 7 = 21 + 8$

**Step 1:** $9x - 7 = 29$ Combine like terms on both sides of the equation.

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$$9x - 7 = 29$$

**Step 2:** $\underline{\quad +7 \quad +7}$ Solve the two-step algebra problem as explained before.

$$9x \quad = 36$$

**Step 3:** $\frac{9x}{9} = \frac{36}{9}$

**Answer:** $x = 4$

**Solve the following problems. Show your work. (DOK 2)**

1. $3a + 1 = a + 9$
2. $9w - 7 = 12w - 13$
3. $-17b + 23 = -4 - 8b$
4. $k + 5 = 20 - 2k$
5. $5 + 16w = 6w - 45$
6. $-12x + 14 = 8x - 46$
7. $p - 1 = 4p + 17$
8. $4d + 14 = 3d - 1$
9. $17 - 9m = m - 23$

## 6.3 Multi-Step Equations (DOK 2)

As we have seen, algebra problems can contain fractions. Some problems will have a fraction that contains a variable as the denominator. You must rationalize the denominator as we learned previously.

**Example 1:** Solve $\frac{1}{x} + 5 = 7$ for $x$.

**Step 1:** The first thing to do is get all of the constants on one side of the equation. Do this by subtracting 5 from both sides.

$$\frac{1}{x} + 5 - 5 = 7 - 5$$

$$\frac{1}{x} = 2$$

**Step 2:** Next, multiply both sides of the equation by $x$ to get $x$ out the denominator of the fraction.

$$\frac{1}{x} \times x = 2 \times x$$

$$1 = 2x$$

**Step 3:** Last, divide both sides by 2 to get $x$ on one side of the equation by itself.

$$\frac{1}{2} = \frac{2x}{2}$$

**Answer:** $\frac{1}{2} = x$ or $x = \frac{1}{2}$


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You can now use what you know about fractions, combining like terms, and solving simple algebra problems to solve problems that involve three or more steps. Study the example below to see how easy it is to solve multi-step problems.

**Example 2:** $3(x + 6) = 5x - 2$

**Step 1:** Use the distributive property to remove parentheses.

$$3x + 18 = 5x - 2$$

**Step 2:** Subtract $5x$ from each side to move the terms with variables to the left side of the equation.

$$\begin{array}{rr} -5x & -5x \\ \hline -2x + 18 & = -2 \end{array}$$

**Step 3:** Subtract 18 from each side to move the integers to the right side of the equation.

$$\begin{array}{rr} & -18 \quad -18 \\ \hline -2x & = -20 \end{array}$$

**Step 4:** Divide both sides by $-2$ to solve for $x$.

$$\frac{-2x}{-2} = \frac{-20}{2}$$

$$x = 10$$

**Solve the following problems. Show your work. (DOK 2)**

1. $\frac{5}{p} + 5 = -10$

2. $\frac{12y - 18}{6} = 4y + 3$

3. $-8(y + 4) = 10y + 4$

4. $\frac{10(x - 2)}{5} = 14$

5. $\frac{12}{x} - 1 = 2$

6. $\frac{x + 4}{3} = 6 - x$

7. $7 - 5x = 10 - (6x + 7)$

8. $-3(x - 4) + 5 = -2x - 2$

9. $\frac{-4x + 3}{2x} = \frac{7}{2x}$

## 6.4 Manipulating Equations (DOK 2)

Sometimes an equation has two variables and you may be asked to solve for one of the variables.

**Example 1:** If $5x + y = 19$, then $y =$

**Answer:** The goal is to have only $y$ on one side of the equation and the rest of the terms on the other side of the equation. Follow order of operations to solve.

$5x + y - 5x = 19 - 5x$ Subtract $5x$ from both sides of the equation.

$y = 19 - 5x$

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Sometimes you are given a formula such as $A = l \times w$ ($A$ = area, $l$ = length, and $w$ = width) and you need to solve a different variable, like $w$.

**Example 2:** The area of a playground is 4500 square feet. The length is 150 feet. What is the width of the playground?

**Step 1:** Starting with $A = l \times w$, you need to solve for $w$. You need to have $w$ on one side of the equation and all the other variables on the other.

$$A = l \times w \Rightarrow \frac{A}{l} = \frac{l \times w}{l} \Rightarrow \frac{A}{l} = w \Rightarrow w = \frac{A}{l}$$

**Step 2:** Now, insert the terms for the formula.

$$w = \frac{4500}{150} = 30$$

You have solved $A = l \times w$ for $w$.

**Solve each of the equations below for the variable indicated. Be sure to follow order of operations. (DOK 2)**

1. If $4a + b = 12$, then $a =$

2. If $6c - d = 17$, then $d =$

3. If $8m - 9n - 2m = 6n$, then $m =$

4. If $-10y - 3x - x = -12y$, then $x =$

5. If $5x - 7y + 9y = -9x + 3$, then $y =$

6. If $-10b + 7a + 3a = -4b$, then $a =$

**Solve each of the following formulas and equations for the given variable. (DOK 2)**

7. $C = 2\pi r$ for $r$

8. $A = \frac{1}{2}bh$ for $h$

9. $V = IR$ for $R$

10. $V = \frac{1}{3}Bh$ for $B$

11. $S = 4\pi r^2$ for $r$

12. $y = mx + b$ for $m$



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## 6.5 Equations with Radicals (DOK 2)

Some multi-step equations contain radicals. An example of a radical is a square root, $\sqrt{\ }$. Work these type of equations out similarly to the section on equations.

**Example 1:** Solve the following equation for $x$. $\sqrt{4x-3}+2=5$

**Step 1:** The first step is to get the constants that are not under the radical on one side. Subtract 2 from both sides of the equation.

$\sqrt{4x-3}+2-2=5-2$

$\sqrt{4x-3}+0=3$

$\sqrt{4x-3}=3$

**Step 2:** Next, you must eliminate the radical sign by squaring both sides of the equation.

$(\sqrt{4x-3})^2=(3)^2$

$4x-3=9$

**Step 3:** Add 3 to both sides of the equation to get the constants on just one side of the equation.

$4x-3+3=9+3$

$4x=12$

**Step 4:** Last, get $x$ on one side of the equation by itself by dividing both sides by 4.

$$\frac{4x}{4}=\frac{12}{4}$$

**Answer:** $x=3$

**Note:** When solving equations with radicals, you must always check for extraneous solutions. In other words, some solutions might not work once you check your answer. To check for extraneous solutions, plug your answer back into the original equation and solve.

Here is the original equation from Example 1 above: $\sqrt{4x-3}+2=5$

**Step 1:** Plug in your answer for $x$.

$\sqrt{4(3)-3}+2=5$

**Step 2:** Solve the equation as normal. Remember to follow the order of operations.

$\sqrt{12-3}+2=5$

$\sqrt{9}+2=5$

$-2\quad -2$

$3$

$\sqrt{9}=3$

$3=3$ This answer checks out. So this is not an extraneous solution.

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**Solve the following equations. Show your work. (DOK 2)**

1. $\sqrt{x} - 13 = -8$
2. $\sqrt{f} + 2 = 8$
3. $5 = \sqrt{6g - 5} + (-2)$
4. $2 = \sqrt{x - 3}$
5. $\sqrt{-d+1} - 9 = -6$
6. $19 - \sqrt{7x - 5} = 16$

**Follow the instructions within each problem below. Show your work. (DOK 3)**

7. Mallory received her test back on radical equations. She missed the last problem: $-\sqrt{x+3} = 4$. The steps she followed are shown below.

**Step 1:** $\dfrac{-1\sqrt{x+3}}{-1} = \dfrac{4}{-1}$

$\sqrt{x+3} = -4$

**Step 2:** $(\sqrt{x+3})^2 = (-4)^2$

$x + 3 = 16$

**Step 3:** $x + 3 - 3 = 16 - 3$

**Answer:** $x = 13$

After looking over her work, Mallory was still convinced that she has the correct answer. She asked to see the teacher's answer key and found the key states that the problem has no solution. Which is correct, Mallory's answer or the teacher's answer key? Create an argument for either Mallory or the answer key and justify your response mathematically.

## 6.6 Multi-Step Inequalities (DOK 2)

Remember that adding and subtracting with inequalities follow the same rules as equations. When you multiply or divide both sides of an inequality by the same positive number, the rules are also the same as for equations. However, when you **multiply or divide** both sides of an inequality by a **negative** number, you must **reverse** the inequality symbol.

**Example 1:** $-x > 4$ — **Remember:** If a number is not in front of a variable, it is assumed to be a 1.

**Step 1:** $\dfrac{x}{-1} > \dfrac{4}{-1}$ — Divide both sides by –1.

**Step 2:** $x < -4$ — Because you divided by a negative, reverse the sign.

Solve multi-step inequalities the same way as multi-step equations. First, add and subtract to isolate the term with the variable. Then, multiply and divide.


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**Example 2:** $2x - 8 > 4x + 1$

| | | |
|---|---|---|
| **Step 1:** | $2x - 8 + 8 > 4x + 1 + 8$<br>$2x > 4x + 9$ | Simplify. Reverse. |
| **Step 2:** | $2x - 4x > 4x + 9 - 4x$<br>$-2x > 9$ | Subtract $4x$ from both sides. |
| **Step 3:** | $\frac{-2x}{-2} > \frac{9}{-2}$ | Divide by $-2$. |
| **Step 4:** | $x < -\frac{9}{2}$ | Remember, change the inequality sign's direction. |

**Solve each of the following inequalities. Show your work. (DOK 2)**

1. $3(2x \quad 5) \geq 8x - 5$
2. $7 + 3y > 2y - 5$
3. $-4x + 2x \geq -8$
4. $-\frac{x}{4} > 12$
5. $-\frac{1}{2}x + 2 > 9$
6. $7 + 4c < -2$

**Follow the instructions within each problem below. Show your work. (DOK 3)**

7. Clive has \$900 in his bank account at the beginning of the school semester. Clive is hoping to have at least \$300 in his bank account at the end of the semester (18 weeks). If Clive only withdraws \$35 a week for food, gas, and entertainment, will Clive be able to reach his goal at the end of the semester? Write an inequality that models the situation, and justify your response both in a written format and algebraically.

8. Brett decides to order some new books using a \$50 gift card he received for his birthday. He decides to buy as many of a new series of books he can without exceeding the amount on his gift card. If each book cost \$12.99 and shipping charges are \$7.99 for any size order, then how many books can Brett purchase in one order?

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## 6.7 Compound Inequalities (DOK 2)

Inequalities can have more than one inequality sign. These are called **compound inequalities**.

**Example 1:** $-2 \leq x < 4$ is read "$-2$ is less than or equal to $x$ and $x$ is less than 4."

You learned earlier in this chapter that you can perform any operation on an equation, as long as you perform the same operation to the other side. The same is true of inequalities. It's also true of compound inequalities.

**Example 2:** Solve the compound inequality $-20 < 5x - 10 < 20$ for $x$.

**Step 1:** We must isolate $x$. The first step is to add 10 to each part of the compound inequality.

$$\begin{array}{rrrrr} -20 & < & 5x-10 & < & 20 \\ +10 & & +10 & & +10 \\ \hline -10 & < & 5x & < & 30 \end{array}$$

**Remember:** What you do to the middle, you must do to the left side and the right side.

**Step 2:** Now divide each part by 5.

$$\frac{-10}{5} < \frac{5x}{5} < \frac{30}{5}$$

**Answer:** $-2 < x < 6$

**Example 3:** Solve the compound inequality $8x + 10 \leq 24$ or $5x + 27 \geq 66$.

**Step 1:** This compound inequality can be solved as two independent inequalities. First, we must isolate $x$. To do this, subtract 10 from both sides of the first inequality and subtract 27 from both sides of the second.

$$\begin{array}{rrrrrrr} 8x+10 & \leq & 24 & \text{or} & 5x+27 & \geq & 66 \\ -10 & & -10 & & -27 & & -27 \\ \hline 8x & \leq & 14 & \text{or} & 5x & \geq & 39 \end{array}$$

**Step 2:** Now, divide both sides of the first inequality by 8 and both sides of the second inequality by 5.

$$\frac{8x}{8} \leq \frac{14}{8} \quad \text{or} \quad \frac{5x}{5} \geq \frac{39}{5}$$

**Answer:** $x \leq 1\frac{3}{4}$ or $x \geq 7\frac{5}{4}$



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**Solve the inequalities. Show your work. (DOK 2)**

1. $-14 < 2x + 4 \leq 14$
2. $6 > 4b - 8 > 0$
3. $-36 < 12e + 3 < 36$
4. $48 > 10c - 12 > 23$
5. $-2 < 9e + 7 \leq 11$
6. $-4 \leq 3g - 40 \leq 38$
7. $35 > 9q - 10 > -28$
8. $2x + 3 \leq 7$ or $6x - 13 > 9$
9. $\frac{x+7}{3} < 4$ or $\frac{2x-6}{2} > 16$

## Chapter 6 Review

**Solve each of the following equations. Show your work. (DOK 2)**

1. $4a - 8 = 28$
2. $-7 + 23w = 108$
3. $\frac{y-8}{6} = 7$
4. $\frac{b+9}{12} = -3$

**Solve. Show your work. (DOK 2)**

5. $19 - 8d = d - 17$
6. $7w - 8w = -4w - 30$
7. $w + 11 + 14 = 15$
8. $5k + 1 - 3 = 11$

**Solve for the variable. (DOK 2)**

9. If $7a + 2b = 1$, then $b =$
10. $h = 6a + 9c^2$ for $c$

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**Solve each of the following equations and inequalities. Show your work. (DOK 2)**

11. $\frac{-11c-35}{4}=4c-2$

12. $7-3x \leq 6x-2$

13. $\frac{5(n+4)}{3}=n-8$

14. $2(3x-1) \geq 3x-7$

**Solve the following problems. Show your work. (DOK 2)**

15. $\frac{12}{x}-5=1$

16. $4+\frac{t}{5}=6$

17. $\frac{24}{p}+3=-3$

**Solve the following problems. Show your work. Check your answers. (DOK 2)**

18. $\sqrt{x}-7=-1$

19. $9+\sqrt{8t-7}=12$

20. $\sqrt{3n+9}+4=-2$

**Solve the following problems. Show your work. (DOK 2)**

21. $-28 < 4x+4 \leq -8$

22. $11 > 2b-6 > 0$



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# Chapter 7
# Linear Equations and Inequalities

Standards covered: A.CED.A.1, A.CED.A.2, A.REI.D.10, A.REI.D.12, F.IF.C.7a

## 7.1 Forms of Linear Equations (DOK 1)

Any equation with two variables that are both to the first power is called a **linear equation**. For example, $x + 3y = 6$ is called "linear" because the graph of the equation is a straight line on a Cartesian plane. Linear equations come in a number of forms.

The **standard form** of a linear equation is $Ax + By = C$, where $A$, $B$, and $C$ are real numbers.

**Examples:** $3x + 2y = 6$ $\quad \frac{6}{5}x + \frac{1}{2}y = -\frac{2}{7}$ $\quad 3.4x - 6.8y = 39.1$ $\quad -x - y = 0$

The **general form** of a linear equation is $Ax + By + C = 0$, where $A$, $B$, and $C$ are real numbers.

**Examples:** $3x + 2y - 6 = 0$ $\quad \frac{6}{5}x + \frac{1}{2}y + \frac{2}{7} = 0$ $\quad 3.4x - 6.8y - 39.1 = 0$ $\quad -x - y = 0$

The **slope-intercept form** of a linear equation is $y = mx + b$, where $m$ is the slope of the line, and $b$ is the $y$-intercept (the point where the line intersects the $y$-axis, $x = 0$).

**Examples:** $y = \frac{3}{2}x - 3$ $\quad y = -\frac{12}{5}x - \frac{4}{7}$ $\quad y = 0.5x - 5.75$ $\quad y = -x$

Let's take a look at converting between forms.

**Example 1:** $4x - y = 8$ is in standard form.

**Step 1:** Subtract 8 from each side: $4x - y - 8 = 8 - 8$. $\Rightarrow 4x - y - 8 = 0$.
Now the equation is in general form.

**Step 2:** Solve for $y$ by adding $y$ to both sides: $4x - y + y - 8 = 0 + y$

**Answer:** $4x - 8 = y$ is now in slope-intercept form.

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**Identify the form of the equation of each line. (DOK 1)**

1. $x + y = 6$
2. $0 = x - 2y + 1$
3. $y = 7x - 2$

## 7.2 Graphing Linear Equations (DOK 2)

In addition to graphing ordered pairs, the Cartesian plane is used to graph the solution set for an equation.

**Example 1:** Graph the solution set for $x + y = 7$.

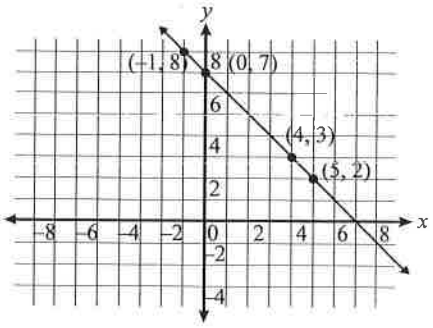
**Step 1:** Choose four random values for $x$ and then find $y$. In this problem, we chose, –1, 0, 4, and 5.

| $x$ | $x + y = 7$ | $y$ | |
|---|---|---|---|
| –1 | $-1 + y = 7$ | 8 | (–1, 8) |
| 0 | $0 + y = 7$ | 7 | (0, 7) |
| 4 | $4 + y = 7$ | 3 | (4, 3) |
| 5 | $5 + y = 7$ | 2 | (5, 2) |

} ordered pair solutions

**Step 2:** Plot these points on a Cartesian plane.

**Step 3:** By passing a line through these points, we graph the solution set for $x + y = 7$. This means that every point on the line is a solution to the equation $x + y = 7$. For example, the line passes through the point (1, 6). The point (1, 6) is a solution for the equation. Just as there are an infinite number of points on a line, there are an infinite number of solutions to the equation of a line.




**Find pairs of numbers that satisfy the equations below and graph the line on graph paper. (DOK 2)**

1. $y = -2x + 2$
2. $-x + 3 = y$
3. $y = \frac{1}{2}x + 1$
4. $4x - 2 = y$
5. $x = 4y - 3$
6. $2x = 3y + 1$



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## 7.3 Graphing Horizontal and Vertical Lines (DOK 2)

The graph of some equations is a horizontal or a vertical line.

**Example 1:** $y = 3$

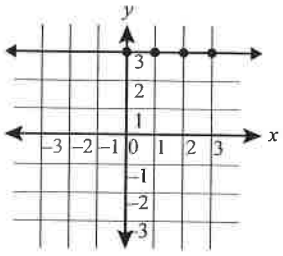
**Step 1:** Make a list of ordered pairs that satisfy the equation $y = 3$.

| $x$ | $y$ |
|---|---|
| 0 | 3 |
| 1 | 3 |
| 2 | 3 |
| 3 | 3 |

No matter what value of $x$ you choose, $y$ is always 3.

**Step 2:** Plot these points on an Cartesian plane and draw a line through the points.

The graph is a horizontal line.




**Example 2:** $2x + 3 = 0$

**Step 1:** For these equations with only one variable, find what $x$ equals first.

$2x + 3 = 0$

$2x = -3$

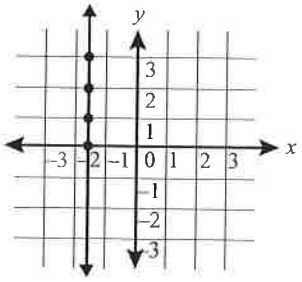
$x = -\frac{3}{2}$

**Step 2:** Just like the example above, find ordered pairs that satisfy the equation, plot the points, and graph the line.

| $x$ | $y$ |
|---|---|
| $-\frac{3}{2}$ | 0 |
| $-\frac{3}{2}$ | 1 |
| $-\frac{3}{2}$ | 2 |
| $-\frac{3}{2}$ | 3 |

No matter which value of $y$ you choose, the value of $x$ does not change.

The graph is a vertical line.




**Find 2 pairs of numbers that satisfy the equations below and graph the line on graph paper. (DOK 2)**

1. $2y - 6 = 0$
2. $x = -3$
3. $4x - 16 = 0$
4. $2y = 1$
5. $3x + 12 = 0$
6. $-4x = 8$
7. $10y + 5 = 0$
8. $-5y = 20$

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## 7.4 Finding the Intercepts of a Line (DOK 2)

The $x$-intercept is the point where the graph of a line crosses the $x$-axis. The $y$-intercept is the point where the graph of a line crosses the $y$-axis.

**To find the $x$-intercept, set $y = 0$ and solve for $x$.**
**To find the $y$-intercept, set $x = 0$ and solve for $y$.**

**Example 1:** Find the $x$- and $y$-intercepts of the line $6x + 2y = 18$.

**Step 1:** To find the $x$-intercept, set $y = 0$ and solve for $x$.

$$6x + 2(0) = 18$$

$$\frac{6x}{6} = \frac{18}{6}$$

$$x = 3$$

The $x$-intercept is at the point (3, 0).

**Step 2:** To find the $y$-intercept, set $x = 0$ and solve for $y$.

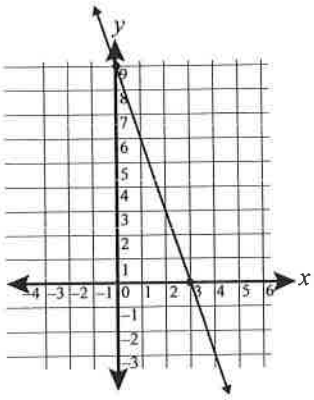
$$6(0) + 2y = 18$$

$$\frac{2y}{2} = \frac{18}{2}$$

$$y = 9$$

The $y$-intercept is at the point (0, 9).

**Step 3:** You can now use the two intercepts to graph the line.




**Find the $x$- and $y$-intercept of each line given. Then graph the line on your own graph paper. (DOK 2)**

1. $7y + 2x = 14$
2. $y + 4 = x$
3. $-8x - 16 = 4y$
4. $9y - 3x = 18$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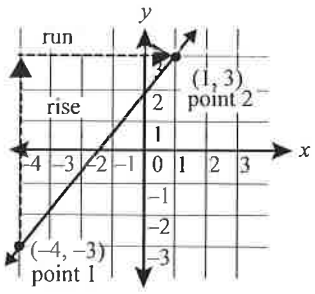
## 7.5 Slope (DOK 2)

The slope of a line, $m$, refers to how steep a line is. When we graph a line using ordered pairs, we can determine the slope.

> The formula for the slope of a line is: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ or $\dfrac{\text{rise}}{\text{run}}$

**Example 1:** What is the slope of the following line that passes through the ordered pairs $(-4, -3)$ and $(1, 3)$?




$y_2$ is 3, the $y$-coordinate of point 2.

$y_1$ is $-3$, the $y$-coordinate of point 1.

$x_2$ is 1, the $x$-coordinate of point 2.

$x_1$ is $-4$, the $x$-coordinate of point 1.

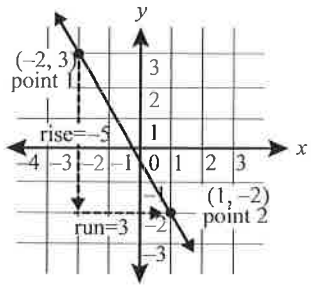
Use the formula for slope given above: $m = \dfrac{3-(-3)}{1-(-4)} = \dfrac{6}{5}$

The slope is $\frac{6}{5}$. This shows us that we can go up 6 (rise) and over 5 to the right (run) to find another point on the line.

**Example 2:** Find the slope of a line through the points $(-2, 3)$ and $(1, -2)$. It doesn't matter which pair we choose for point 1 and point 2. The answer is the same.

Let point 1 be $(-2, 3)$.
Let point 2 be $(1, -2)$.

$$\text{slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{-2-3}{1-(-2)} = \frac{-5}{3}$$

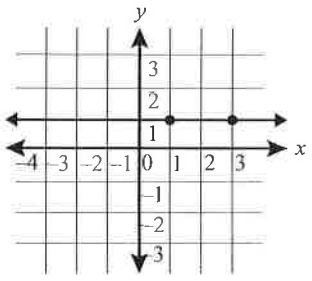



When the slope is negative, the line will slant down from left to right. For this example, the line will go **down** 5 units and then over 3 units to the **right**.

**Example 3:** What is the slope of a line that passes through (1, 1) and (3, 1)?

$$\text{slope} = \frac{1-1}{3-1} = \frac{0}{2} = 0$$

When $y_2 - y_1 = 0$, the slope will equal 0, and the line will be horizontal.

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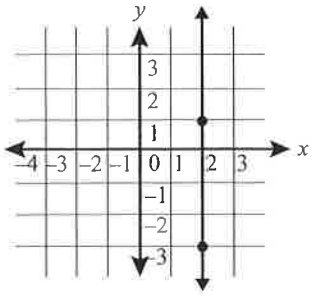
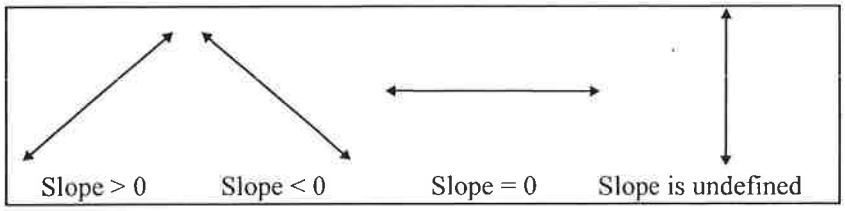
**Example 4:** What is the slope of a line that passes through (2, 1) and (2, −3)?

$$\text{slope} = \frac{-3-1}{2-2} = \frac{-4}{0} = \text{undefined}$$




Remember, you cannot divide by zero. When $x_2 - x_1 = 0$, the slope is undefined, and the line will be vertical.

**The following lines summarize what we know about slope.**




**Find the slope of the line that goes through the following pairs of points. (DOK 2) (See section examples for slope formula.)**

1. (2, 3) (4, 5)
2. (−1, 2) (4, 1)
3. (3, 0) (3, 4)
4. (3, 2) (−1, 8)
5. (6, −2) (3, −2)
6. (1, 2) (3, 4)

## 7.6 Slope-Intercept Form of a Line (DOK 2)

The graph for a linear equation is a straight line. To put a linear equation in slope-intercept form, solve the equation for $y$. This form of the equation shows the slope and the $y$-intercept. Slope-intercept form follows the pattern of $y = mx + b$. The $m$ represents slope, and the $b$ represents the $y$-intercept.

**Example 1:** Put the equation $2x + 3y = 15$ in slope-intercept form. What is the slope of the line? What is the $y$-intercept? Graph the line.

**Step 1:** Solve for $y$:

$$\begin{array}{rcr} 2x + 3y & = & 15 \\ -2x & & -2x \\ \hline \frac{3y}{3} & = & -\frac{2x}{3} + \frac{15}{3} \end{array}$$

slope-intercept form: $y = -\frac{2}{3}x + 5$

The slope is $-\frac{2}{3}$ and the $y$-intercept is 5.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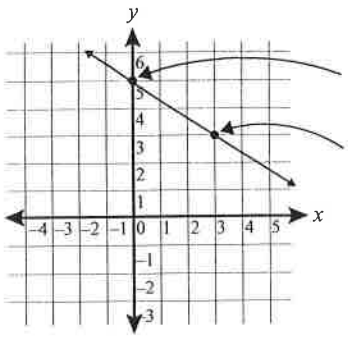
**Step 2:** Using the slope and the $y$-intercept, we can graph the line. The $y$-intercept is the first point to plot on the graph.




The $y$-intercept is 5, so the line passes through the point (0, 5) on the $y$-axis.

Use the slope, $-\frac{2}{3}$, as a guide to get a second point.

**Put each of the following equations in slope-intercept form by solving for $y$. On your graph paper, graph the line using the slope and $y$-intercept. (DOK 2)**

1. $4x - y = 5$
2. $x + 3y = -12$
3. $2x = 4y - 2$
4. $6x - 3y = 9$
5. $4x + 2y = 8$
6. $-2x - 4y = 8$

## 7.7 Writing the Equation of a Line Given a Point and Slope (DOK 2)

If you are given the slope of a line, $m$, and the coordinates of one point, you can write the equation of a line. Using the point-slope form of an equation, $y - y_1 = m(x - x_1)$, substituting the given data and solving for $y$ will put an equation in slope-intercept form.

**Example 1:** Write the equation of a line that contains the point (3, –1) and has a slope of $\frac{4}{3}$.

**Step 1:** Using $y - y_1 = m(x - x_1)$, substitute the given data into the question. $(x, y)$ is represented by (3, –1) and $m = \frac{4}{3}$.

$$y-(-1)=\frac{4}{3}(x-3)$$

$$y+1=\frac{4}{3}(x-3)$$

**Step 2:** Solve for $y$.

$$y+1=\frac{4}{3}(x-3)$$

$$y+1=\frac{4}{3}x-4$$

$$\underline{\quad -1= \quad -1}$$

$$y \quad =\frac{4}{3}x-5$$

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**Write the equation of a line given the point and slope. (DOK 2)**

1. $(3, -4)$, $m = \frac{1}{2}$
2. $(-2, 1)$, $m = \frac{4}{3}$
3. $(-1, 5)$, $m = \frac{2}{5}$
4. $(-2, 5)$, $m = \frac{1}{3}$
5. $(-2, -3)$, $m = -\frac{3}{4}$
6. $(2, -4)$, $m = 1$

## 7.8 Writing the Equation of a Line Given Two Points (DOK 2)

If you are given the coordinates of 2 points, you can write the equation of a line. You must first find the slope. Then, you may pick either coordinate and substitute it into point-slope form. Both points will work if solved correctly.

**Example 1:** Write the equation of the line passing through the points $(-2, 3)$ and $(1, 5)$.

**Step 1:** First, find the slope of the line using the two points given.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{1-(-2)} = \frac{2}{3}$$

**Step 2:** Pick one of the two points to use in the point-slope equation. For point $(-2, 3)$, we know $x_1 = -2$ and $y_1 = 3$, and we know $m = \frac{2}{3}$. Substitute these values into the point-slope form of the equation.

$$y - y_1 = m\,(x - x_1)$$

$$y - 3 = \tfrac{2}{3}[x - (-2)]$$

$$y - 3 = \tfrac{2}{3}x + \tfrac{4}{3}$$

$$y = \tfrac{2}{3}x + \tfrac{13}{3}$$

Choose the other coordinate to check your answer.

**Step 1:** Substitute the other coordinate into the equation.

$$y - 5 = \frac{2}{3}(x - 1)$$

$$y - 5 = \frac{2}{3}x - \frac{2}{3}$$

$$\begin{array}{rr} +5 & +5 \\ \hline \end{array}$$

$$y = \frac{2}{3}x + \frac{13}{3}$$



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You can write equations given the correct data. If the increases or decreases in the ordered pairs are at a constant rate, then a linear equation can be calculated for the data.

**Example 2:** Write an equation for the following set of data.
Dan set his car on cruise control and noted the distance he went every 5 minutes.

| Minutes in operation ($x$) | Odometer reading ($y$) |
|---|---|
| 5 | 28,490 miles |
| 10 | 28,494 miles |

**Step 1:** Write two ordered pairs in the form (minutes, distance) for Dan's driving, (5, 28490) and (10, 28494), and find the slope, or the rate of change.

$$m = \frac{28494 - 28490}{2} = \frac{4}{5}$$

**Step 2:** Use the ordered pairs to write the equation in the form $y = mx + b$. Place the slope, $m$, and one of the pairs of points as $x_1$ and $y_1$ in the formula, $y - y_1 = m\,(x - x_1)$.

$$y - 28490 = \tfrac{4}{5}(x - 5)$$

$$y - 28490 = \tfrac{4}{5}x - 4$$

$$y - 28490 + 28490 = \tfrac{4}{5}x - 4 + 28490$$

$$y = \tfrac{4}{5}x + 28486$$

As you can see, there are many different ways to find equations using other information. Let's look at how a table of values can be used to determine equations.

**Example 1:** Use the chart below to find the equation of the values.

| $x$ | $-2$ | $-1$ | $1$ | $2$ |
|---|---|---|---|---|
| $y$ | $-7$ | $-4$ | $2$ | $5$ |

**Step 1:** Unfortunately, the $y$-intercept cannot always be found by looking at the chart. The $y$-intercept is not shown in the chart, so it must be found algebraically.

**Step 2:** Find the slope first. Use the points $(-1, -4)$ and $(1, 2)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-4)}{1-(-1)} = \frac{6}{2} = 3$$

The slope is 3.

**Step 3:** After you find the slope, pick a random point from the table. Substitute the slope for $m$ and the random point for $x_1$ and $y_1$ in the following equation.

$y - y_1 = m\,(x - x_1)$

For this example, let's choose (2, 5).

$y - 5 = 3(x - 2)$

**Step 4:** Simplify.

$y - 5 = 3x - 6$

**Answer:** $y = 3x - 1$

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**Write an equation of a line passing through the two points. (DOK 2)**

1. $(-1, -4)$ and $(2, -1)$

2. $(-2, 5)$ and $(-4, 3)$

3. $(0, -2)$ and $(6, -2)$

4. $(3, -4)$ and $(-3, 4)$

**Write an equation for each of the following sets of data, assuming the relationship is linear. (DOK 3)**

5. **Doug's Doughnut Shop**

| Year in Business | Total Sales |
|---|---|
| 1 | \$55,000 |
| 4 | \$85,000 |

6.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | −24 | −44 | −64 | −84 | −104 |

## 7.9 Changing the Slope or Y-Intercept of a Line (DOK 3)

When the slope and/or the $y$-intercept of a linear equation changes, the graph of the line will also change.

**Example 1:** Consider line $l$ shown in Figure 1 at right. What happens to the graph of the line if the slope is changed to $\frac{4}{5}$?

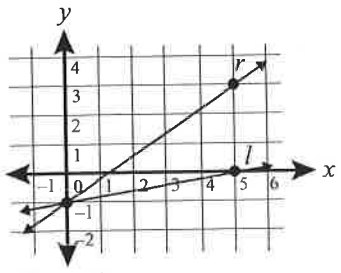

Figure 1

**Step 1:** Determine the $y$-intercept of the line. For line $l$, the $y$-intercept is at the point $(0, -1)$.

**Step 2:** Find the slope of the line using two points that the line goes through: $(0, -1)$ and $(5, 0)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-1)}{5-0} = \frac{1}{5}$$

**Step 3:** Write the equation of line $l$ in slope-intercept form:

$y = mx + b \quad \Longrightarrow \quad y = \frac{1}{5}x - 1$

**Step 4:** Rewrite the equation of the line using a slope of $\frac{4}{5}$ and then graph the line. The equation of the new line is $y = \frac{4}{5}x - 1$.

**Answer:** The graph of the new line is labeled line $r$ and is shown in Figure 1. A line with a slope of $\frac{4}{5}$ is steeper than a line with a slope of $\frac{1}{5}$.


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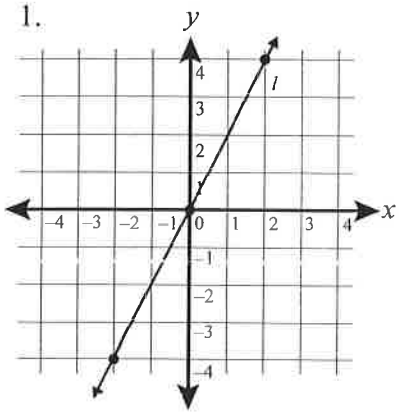
**Consider the line ($l$) shown on each of the following graphs, and write the equation of the line in the space provided. Then, on the same graph, graph the line ($r$) for which the equation is given. Write how the slope and $y$-intercept of line $l$ compare to the slope and $y$-intercept of line $r$ for each graph. (DOK 3)**

1.



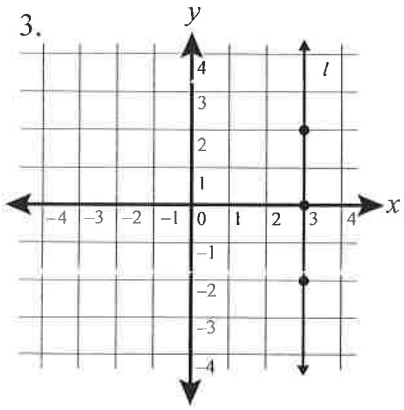

line $l$: _____

line $r$: $y = -2x$

slopes: _____

$y$-intercepts: _____

3.



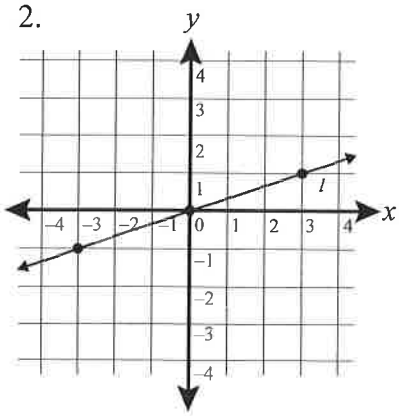

line $l$: _____

line $r$: $y = -3$

slopes: _____

$y$-intercepts: _____

2.



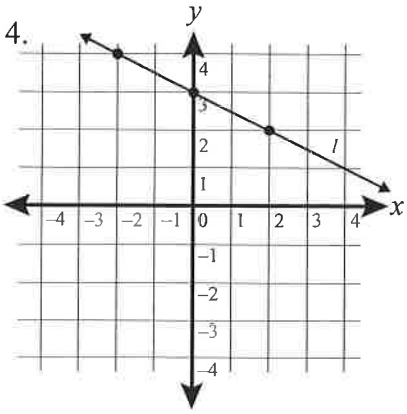

line $l$: _____

line $r$: $y = \frac{1}{3}x + 2$

slopes: _____

$y$-intercepts: _____

4.




line $l$: _____

line $r$: $y = -\frac{1}{2}x - 3$

slopes: _____

$y$-intercepts: _____

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## 7.10 Graphing Linear Data (DOK 2, 3)

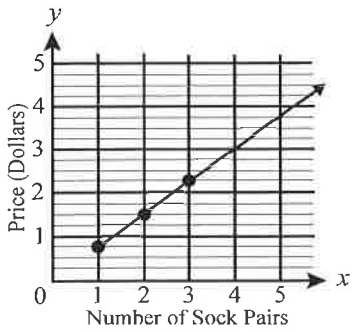
We relate many types of data by a constant ratio. As you learned on the previous page, this type of data is linear. The **slope**, or **rate of change**, of the line described by linear data is the ratio between the data. Plotting linear data with a constant ratio can be helpful in finding additional values. Often linear data is not negative because many objects cannot be measured in negative values. When there are no negative values for these type of problems, the actual graph will be a ray instead of a line.

**Example 1:** A department store prices socks per pair. Each pair of socks costs \$0.75. Plot pairs of socks versus price on a Cartesian plane.

**Step 1:** Since the price of the socks is constant, you know that one pair of socks costs \$0.75, 2 pairs of socks cost \$1.50, 3 pairs of socks cost \$2.25, and so on. Make a list of a few points.

| Pair(s) $x$ | Price $y$ |
|---|---|
| 1 | 0.75 |
| 2 | 1.50 |
| 3 | 2.25 |

**Step 2:** Plot these points on a Cartesian plane, and draw a straight line through the points.




**Example 2:** What is the slope of the data in the example above? What does the slope describe?

**Solution:** You can determine the slope either by the graph or by the data points. For this data, the slope is 0.75. Remember, slope is rise/run. For every \$0.75 going up the $y$-axis, you go across one pair of socks on the $x$-axis. The slope describes the price per pair of socks.

**Example 3:** Use the graph created in the above example to answer the following questions. How much would 5 pairs of socks cost? How many pairs of socks could you purchase for \$3.00? Extending the line gives useful information about the price of additional pairs of socks.

**Solution:** The line that represents 5 pairs of socks intersects the data line at \$3.75 on the $y$-axis. Therefore, 5 pairs of socks would cost \$3.75. Also, the line representing the value of \$3.00 on the $y$-axis intersects the data line at 4 on the $x$-axis. Therefore, \$3.00 will buy exactly 4 pairs of socks.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**Use the information given to make a line graph for each set of data and answer the questions related to each graph. (DOK 2, 3)**

1. The diameter of a circle versus the circumference of a circle is a constant ratio. Use the data given below to graph a line to fit the data. Extend the line, and use the graph to answer the next question.

**Circle**

| Diameter | Circumference |
|---|---|
| 4 | 12.56 |
| 5 | 15.70 |

2. Using the graph of the data in question 1, estimate the circumference of a circle that has a diameter of 3 inches.

3. What is the slope of the line you graphed in question 1?

4. The length of a side on a square and the perimeter of a square are constant ratios to each other. Use the data below to graph this relationship.

**Square**

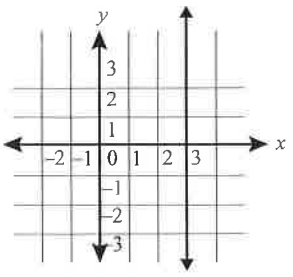
| Length of side | Perimeter |
|---|---|
| 4 | 16 |
| 5 | 20 |

5. Using the graph from question 4, what is the perimeter of a square with a side that measures 2 inches?

6. What is the slope of the line graphed in question 4?

## 7.11 Graphing Inequalities (DOK 2)

Here is the graph the equation $x = 3$:

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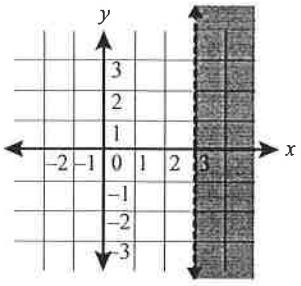
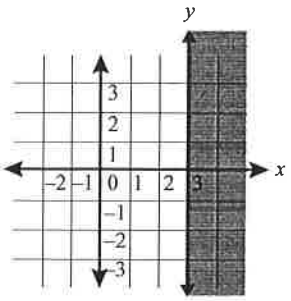
The graph of inequalities such as $x > 3$. Use a broken line since the points on the line $x = 3$ *are not* included in the solution. Shade all points greater than 3.




The graph of $x \geq 3$ uses a solid line because the points on the line $x = 3$ *are* included in the graph.

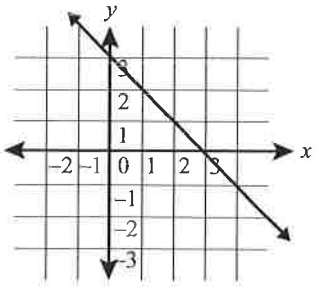



**Example 1:** Graph $x + y \geq 3$.

**Step 1:** First, graph $x + y \geq 3$ by changing the inequality to an equality. Think of ordered pairs that will satisfy the equation $x + y = 3$. Then, plot the points, and draw the line.

As shown below, this line divides the Cartesian plane into 2 half-planes, $x + y \geq 3$ and $x + y \leq 3$. One half-plane is above the line, and the other is below the line.

| $x$ | $y$ |
|---|---|
| 2 | 1 |
| 0 | 3 |
| 3 | 0 |
| 4 | −1 |



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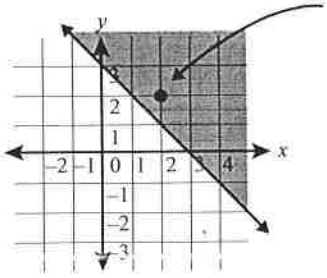
**Step 2:** To determine which side of the line to shade, choose a test point. If the point you choose makes the inequality true, then the point is on the side you shade. If the point you choose does not make the inequality true, then shade the side that does not contain the test point.




For our test point, let's choose (2, 2).
Substitute (2, 2) into the inequality.

$x + y \geq 3$
$2 + 2 \geq 3$

$4 \geq 3$ is true, so shade the side that includes this point.

Use a solid line because of the $\geq$ sign.

For more complex inequalities, it is easier to graph by first changing the inequality to an equality and then put the equation in slope-intercept form.

**Example 2:** Graph the inequality $2x + 4y < 8$.

**Step 1:** Change the inequality symbol to an equal sign.
$2x + 4y = 8$

**Step 2:** Put the equation in slope-intercept form by solving the equation for $y$.

$$\begin{aligned} 2x + 4y &= 8 \\ 2x - 2x + 4y &= -2x + 8 \\ 4y &= -2x + 8 \\ \frac{4y}{4} &= \frac{-2x+8}{4} \\ y &= \frac{-2x}{4} + \frac{8}{4} \\ y &= -\tfrac{1}{2}x + 2 \end{aligned}$$

Subtract $2x$ from both sides of the equation.
Simplify.

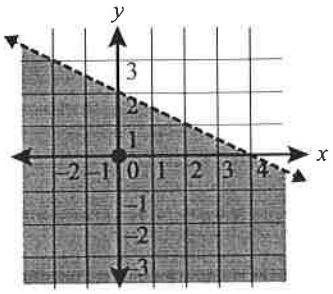
Divide both sides by 4.

Find the lowest terms of the fractions.

**Step 3:** Graph the line. If the inequality is $<$ or $>$, use a dotted line. If the inequality is $\leq$ or $\geq$, use a solid line. For this example, use a dotted line.

**Step 4:** Determine which side of the line to shade. Pick a point such as (0, 0) to see if it is true in the inequality.

$2x + 4y \leq 8$, so substitute (0, 0).
Is $0 + 0 \leq 8$? Yes, $0 \leq 8$, so shade the side of the line that includes the point (0, 0).

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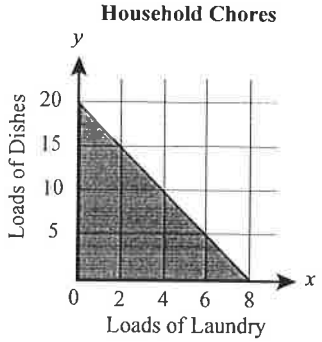
Graphs of linear inequalities can be used to represent and solve real-life problems and situations.

**Example 3:** Becca has at most 5 hours to finish washing the dishes and doing several loads of laundry. The graph shows different combinations of dishes and loads of laundry she can wash.




Can Becca wash 5 loads of dishes and 4 loads of laundry in 5 hours?

We can see by looking at the graph that the $x$-axis represents loads of laundry and the $y$-axis represents the loads of dishes. First, find 4 on the $x$-axis and 5 on the $y$-axis. Then, find if the point where they meet is inside or outside of the shaded area. The point is within the shaded area; therefore, Becca can wash 5 loads of dishes and 4 loads of laundry in 5 hours.

**Graph the following inequalities on your own graph paper. (DOK 2)**

1. $y > -5$
2. $x \leq 0$
3. $x < 4$
4. $x < y + 4$
5. $2x + y \geq 1$
6. $x + y < -1$
7. $-4y \geq 2x + 1$
8. $3x - y \geq 4$
9. $-2y < 4x - 1$

**Use the graph to answer the question below. (DOK 1)**

Rasool has at most 4 hours to finish reading the pages in his government book and writing his research paper for his English class. The graph shows different combinations of pages he can read for government and pages he can write for English.

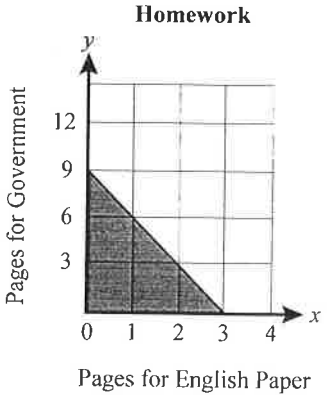
10. Which of these combinations could Rasool not complete in 4 hours?

    **A** 3 English pages, no government pages

    **B** 1 English page, 5 government pages

    **C** 2 English pages, 2 government pages

    **D** 2 English pages, 6 government pages



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## 7.12 Real-World Linear Equations (DOK 2)

Linear equations show relationships between two variables.

**Example 1:** A local cell phone company uses the equation $y = \frac{5}{2}x + 10$ to determine the charges for usage where $y$ = the cost and $x$ = the minutes used. How much will Jessica's bill be if she talked for 40 minutes?

**Step 1:** Substitute the known value for $x$.
$y = \frac{5}{2}(40) + 10$

**Step 2:** Simplify.
$y = 100 + 10 = 110$
Jessica's bill will be \$110.

**Example 2:** Vincent bought a luxury car for \$165,000 and its value has depreciated linearly. After 5 years the value was \$137,000. What is the amount of yearly depreciation?

**Step 1:** First, get points to plug into the slope formula.
(0, 165000) and (5, 137000)
The values for $x$ represent the year the car was bought and 5 years later. The $y$ values represent the two cost values.

**Step 2:** Solve for the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \frac{137,000 - 165,000}{5 - 0} \qquad m = \frac{-28,000}{5} = -5,600$$

The value of Vincent's car depreciated \$5,600 each year.

**Example 3:** In 1990, the average cost of a new house was \$123,000. By the year 2000, the average cost of a new house was \$134,150. Based on a linear model, what is the predicted average cost for 2008?

**Step 1:** First, get points to plug into the slope formula.
(0, 123000) and (10, 134150)
In this example, $x_2$ is the number of years after the start of the data (2000–1990).

**Step 2:** Solve for the slope.

$$m = \frac{134,150 - 123,000}{10 - 0} \qquad m = \frac{11,150}{10} = 1,115$$

The slope is 1,115.

**Step 3:** Next, we can plug the known values into a linear equation. To find the new value for $x$, subtract 2008 – 1990 = 18.
$y = 1,115(18) + 123,000$
$y = 143,070$
\$143,070 is the predicted average cost of a new house for 2008.

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**Solve the following problems. (DOK 2)**

1. Acacia bought an MP3 player at Everywhere Electronics for $350 and its valued depreciated linearly. Three years later, she saw the same MP3 player at Everywhere Electronics for $125. What is the amount of yearly depreciation of Acacia's MP3 player?

2. A small plane costs $500,000 new. Twenty years later it is valued at $150,000. Assuming a linear depreciation, what was the value of the plane when it was 14 years old?

3. The attendance on the first day of the Sunny Day Festival was 325 people. The attendance on the third day was 382 people. Assuming the attendance will increase linearly each day, how many people will attend the Sunny Day Festival on the seventh day?

4. Two years ago Juanita bought 2 shirts for $15 and last year she bought 4 shirts for $45. Assuming the price will increase linearly, how much will 8 shirts cost Juanita this year?

**Follow the directions in each problem below. (DOK 2 and 3)**

5. Abbey is looking for a new cell phone provider. In her search, she has found 3 local companies: Gift of Gab, On the Go, and Connect. To determine their monthly charges, the 3 companies use the following equations, where $y$ is the monthly charge and $x$ is the number of minutes used. .

   Connect: $y = 6x - 100$ Gift of Gab: $y = \frac{3}{4}x + 20$ On the Go: $y = \frac{1}{2}x + 60$

   A) Which is the cheapest provider if Abbey uses 200 minutes per month?

   B) What if she used 100 minutes?

6. The Kramer's family chihuahua, Misty, currently weighs 12 pounds. The veterinarian informed Mrs. Kramer that chihuahuas are supposed to weigh between 8 and 12 pounds. The Doctor put Misty on a diet to lose $\frac{1}{2}$ pound every 4 weeks until she weighs 8 pounds.

   A) Write a linear equation in standard form that models Misty's diet plan.

   B) Graph your equation on your own paper.

   C) Interpret and explain the meaning of the $x$ and $y$ intercept in terms of the problem, if they exist.

   D) If the diet plan is successful, how many weeks will it take Misty to weigh 8 pounds?


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## Chapter 7 Review

**Answer the following questions. Use your own graph paper. (DOK 2)**

1. Graph the line which has a slope of $-2$ and a $y$-intercept of $-3$.

2. Graph the equation $2y = 8$.

3. What is the $x$-intercept for the following equation? $6x - y = 30$

4. What is the $y$-intercept for the following equation? $4x + 2y = 28$

5. Write the following equation in slope-intercept form. $3x = -2y + 4$

6. What is the slope of the line $y = -\frac{1}{2}x + 3$?

7. Write the equation of a line that passes through the points $(-1, 4)$ and $(-6, -2)$.

8. Write the equation of the line which contains the point $(0, 2)$ and has a slope of $\frac{3}{4}$.

**Graph the following inequalities on a Cartesian plane using your graph paper. (DOK 2)**

9. $x \geq 4$

10. $5y > -10x + 5$

**Use the data given to answer the following questions. (DOK 1, 2, 3)**

11. Darien and his family are at a football game. Darien's dad gives him \$22 to buy snacks. He finds that drinks cost \$1.50 and a bowl of nachos cost \$2.75. Write and graph an inequality that shows the amount of sodas and nachos that Darien can afford to buy.

12. Write an equation that fits the data given below. Assume the data is linear.

| Plumber Charges per Hour | |
|---|---|
| Hour | Charge |
| 1 | \$120 |
| 2 | \$170 |

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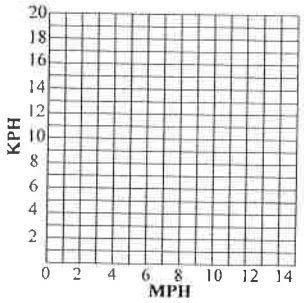
13. The data given below shows conversions between miles per hour and kilometers per hour. Based on this data, graph a conversion line on the Cartesian plane below.

| Speed | |
|---|---|
| MPH | KPH |
| 5 | 8 |
| 10 | 16 |




14. What would be the approximate conversion of 9 mph to kph?

15. Use the data given below to graph the interest rate versus the interest on $80.00 in one year. Use your own graph paper.

| $80.00 Principal | |
|---|---|
| Interest Rate | Interest-1 year |
| 5% | $4.00 |
| 10% | $8.00 |

16. About how much interest would accrue in one year at an 8% interest rate? Justify your response.

17. What is the slope of the line describing interest rate versus interest?

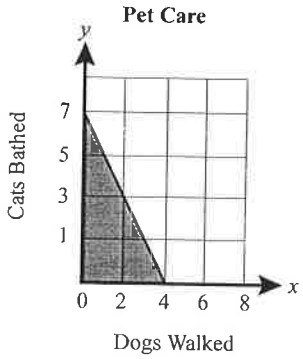
18.

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | 4 | 5 | 6 | 7 | 8 |

19.

| $x$ | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| $y$ | −4 | −3.5 | −3 | −2.5 | −2 |

20. Shemmyla owns a business taking care of cats and dogs. Today, she has at most 6 hours to bathe the cats and walk the dogs. Based on the graph, can Shemmyla bathe 4 cats and walk 5 dogs in 6 hours? Explain your response. (DOK 3)



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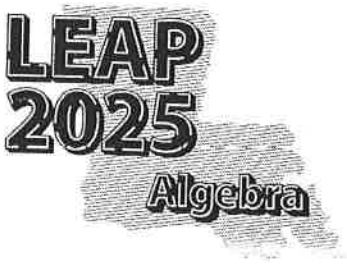
# Chapter 8
# Systems of Equations and Inequalities




Standards covered: A.CED.A.3, A.REI.C.5, A.REI.C.6, A.REI.D.11, A.REI.D.12

## 8.1 Systems of Equations (DOK 2)

Two linear equations considered at the same time are called a **system** of linear equations. The graph of a linear equation is a straight line. The graphs of two linear equations can show that the lines are **parallel**, **intersecting**, or **coinciding**. Two lines that are **parallel** will never intersect and have no ordered pairs in common. If two lines are **intersecting**, they have one point in common, and in this chapter, you will learn to find the ordered pair for that point. If the graph of two linear equations is the same line, the lines are said to be **coinciding**.

If you are given a system of two linear equations, and you put both equations in slope-intercept form, you can immediately tell if the graph of the lines will be parallel, intersecting, or coinciding.

If two linear equations have the same slope and the same $y$-intercept, then they are both equations for the same line, or **coinciding** lines. A line is made up of an infinite number of points extending infinitely far in two directions. Therefore, coinciding lines have an infinite number of points in common.

**Example 1:** $2x + 3y = -3$ **In slope-intercept form:** $y = -\frac{2}{3}x - 1$

$4x + 6y = -6$ **In slope-intercept form:** $y = -\frac{2}{3}x - 1$

**The slope and $y$-intercept of both lines are the same.**

If two linear equations have the same slope but different $y$-intercepts, they are **parallel** lines. Parallel lines never touch each other, so they have no points in common.

If two linear equations have different slopes, then they are **intersecting** lines and share exactly one point in common.

The chart below summarizes what we know about the graphs of two equations in slope-intercept form.

| $y$-intercepts | Slopes | Graphs | Number of Solutions |
|---|---|---|---|
| same | same | coinciding | infinite |
| different | same | distinct parallel lines | none (they never touch) |
| same or different | different | intersecting lines | exactly one |

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**For the pairs of equations below, put each equation in slope-intercept form and tell whether the graphs of the lines will be coinciding, parallel, or intersecting. (DOK 2)**

1. $-x + y = -5$
   $x - y = 5$

2. $3x = y + 1$
   $y = 3x + 1$

3. $-2x + 3y = 5$
   $x = 2 - y$

4. $2x + 2y = 18$
   $y + x = 9$

5. $8y = x - 5$
   $y - \frac{1}{8}x = 12$

6. $3x - y = 1$
   $2y = -6x + 5$

## 8.2 Solving Systems of Equations by Graphing (DOK 2)

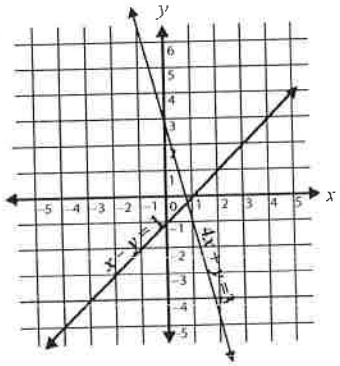
When two lines intersect, they share exactly one point in common.

**Example 1:** $4x + y = 3$ and $x - y = 1$

Put each equation in slope-intercept form.

$4x + y = 3$ $\qquad$ $x - y = 1$
$y = -4x + 3$ $\qquad$ $-y = -x + 1$
$\qquad\qquad\quad$ $y = x - 1$

Straight lines with different slopes are **intersecting lines**. Look at the graphs of the lines on the same Cartesian plane.




**Use your own paper to graph the following equations and determine where the graphs intersect. (DOK 2)**

1. $y = x + 2$
   $y = -7x - 4$

2. $y = 3x + 8$
   $y = -x + 4$

3. $y = 2x - 14$
   $y = -x + 1$

4. $y = 4x - 3$
   $y = 3x + 4$

5. $y = -x$
   $y = \frac{5}{2}x + 7$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## 8.3 Solving Systems of Equations by Substitution (DOK 2)

You can solve systems of equations by using the substitution method.

**Example 1:** Find the point of intersection of the following two equations:

Equation 1: $x - y = 1$

Equation 2: $x + 3y = 9$

**Step 1:** Solve one of the equations for $x$ or $y$. Let's choose to solve Equation 1 for $x$.

Equation 1: $x - y = 1$

$x = y + 1$

**Step 2:** Substitute the value of $x$ from Equation 1 in place of $x$ in Equation 2.

Equation 2: $x + 3y = 9$

$(y + 1) + 3y = 9$

$4y + 1 - 9$

$4y = 9 - 1$

$4y = 8$

$y = 2$

**Step 3:** Substitute the solution for $y$ back into Equation 1 and solve for $x$.

Equation 1: $x - y = 1$

$x - 2 = 1$

$x = 3$

**Step 4:** The solution set is (3, 2). Substitute in one or both of the equations to check.

| Equation 1: | Equation 2: |
|---|---|
| $x - y = 1$ | $x + 3y = 9$ |
| $3 - 2 = 1$ | $3 + 3(2) = 9$ |
| $1 = 1$ | $3 + 6 = 9$ |
| | $9 = 9$ |

**Answer:** The point (3, 2) is common for both equations. This is the **point of intersection.**

**For each of the following pairs of equations, find the point of intersection (common solution) using the substitution method. (DOK 2)**

1. $y - 5 = x$
   $y - 7 = 2x$

2. $y = 2 - x$
   $5 = y + 2x$

3. $y - 9 = -x$
   $-7 = y - 3x$

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4. $5y - 8 = x$
   $3y - 4 = x$

6. $y + 3 = 2x$
   $y + 5 = 3x$

8. $x - y = 0$
   $y + 1 = 2x$

5. $2y + 6 = x$
   $y - 3x = -13$

7. $y + \frac{1}{3}x = -1$
   $y + 1 = 2x$

9. $y + x = -1$
   $y - 2x = -16$

## 8.4 Solving Systems of Equations by Elimination (DOK 2)

You can solve systems of equations algebraically by adding or subtracting an equation from another equation or system of equations.

**Example 1:** Find the point of intersection of the following two equations:

Equation 1: $x + y = 10$

Equation 2: $-x + 4y = 5$

**Step 1:** Make sure to line up the variables. Eliminate one of the variables by adding the two equations together. Since the $x$ has the same coefficient in each equation, but opposite signs, it will cancel nicely by adding.

$$\begin{array}{rl} x + y = 10 & \\ +(-x + 4y = 5) & \\ \hline 0 + 5y = 15 & \text{Simplify.} \\ 5y = 15 & \text{Divide both sides by 5.} \\ y = 3 & \end{array}$$

Add each like term together.

**Step 2:** Substitute the solution for $y$ back into an equation, and solve for $x$.

Equation 1: $x + y = 10$ Substitute 3 for $y$.

$x + 3 = 10$ Subtract 3 from both sides.

$x = 7$

**Step 3:** The solution set is (7, 3). To check, substitute the solution into both of the original equations.

Equation 1: $x + y = 10$

$7 + 3 = 10$

$10 = 10$

Equation 2: $-x + 4y = 5$

$-(7) + 4(3) = 5$

$-7 + 12 = 5$

$5 = 5$

**Answer:** The point (7, 3) is the point of intersection.



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**Example 2:** Find the point of intersection of the following two equations:

Equation 1: $3x - 2y = -1$

Equation 2: $-4y = -x - 7$

**Step 1:** Put the variables in Equation 2 on the same side.

| | |
|---|---|
| $-4y = -x - 7$ | Add $x$ to both sides. |
| $x - 4y = -x + x - 7$ | Simplify. |
| $x - 4y = -7$ | |

**Step 2:** $3x - 2y = -1$
$x - 4y = -7$

Looking above, we cannot combine the two equations to eliminate a variable. We must choose a variable that we want to eliminate. It does not matter which one but the variable you choose has to be the exact opposite.

For this problem, let's choose to eliminate the $y$. In order to do so, we have to multiply the top equation by –2. By doing this, the coefficients of y will be opposite and can be eliminated.

$-2\,(3x - 2y = -1) \rightarrow -6x + 4y = 2$

$x - 4y = -7 \rightarrow x - 4y = -7$

**Step 3:** Add the two equations.

| | |
|---|---|
| $-6x + 4y = 2$ | |
| $+(x - 4y = -7)$ | Add Equation 2 to Equation 1. |
| $-5x + 0 = -5$ | Simplify. |
| $-5x = -5$ | Divide both sides by $-5$. |
| $x = 1$ | |

**Step 4:** Substitute the solution for $x$ back into an equation and solve for $y$.

| | | |
|---|---|---|
| Equation 1: | $3x - 2y = -1$ | Substitute 1 for $x$. |
| | $3\,(1) - 2y = -1$ | Simplify. |
| | $3 - 2y = -1$ | Subtract 3 from both sides. |
| | $3 - 3 - 2y = -1 - 3$ | Simplify. |
| | $-2y = -4$ | Divide both sides by $-2$. |
| | $y = 2$ | |

**Step 5:** The solution set is (1, 2). To check, substitute the solution into both of the original equations.

| | | | |
|---|---|---|---|
| Equation 1: | $3x - 2y = -1$ | Equation 2: | $-4y = -x - 7$ |
| | $3\,(1) - 2\,(2) = -1$ | | $-4\,(2) = -1 - 7$ |
| | $3 - 4 = -1$ | | $-8 = -8$ |
| | $-1 = -1$ | | |

**Answer:** The point (1, 2) is the point of intersection.

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**For each of the following pairs of equations, find the point of intersection using elimination. (DOK 2)**

1. $x + 2y = 8$
   $-x - 3y = 2$

2. $x - y = -1$
   $x + y = 9$

3. $3x - y = -1$
   $x + y = 13$

4. $x + 4y = 10$
   $x + 7y = 16$

5. $2x - y = 2$
   $4x - 9y = -3$

6. $-x = y - 1$
   $x = y - 1$

7. $5x + 2y = 1$
   $4x + 8y = 20$

8. $3x - 2y = 14$
   $x - y = 6$

9. $x = 2y + 3$
   $y = 3 - x$

## 8.5 Solving Systems of Equations Word Problems (DOK 3)

Certain word problems can be solved using systems of equations.

**Example 1:** In a game show, Andre earns 6 points for every right answer and loses 12 points for every wrong answer. He answered correctly 12 times as many questions as he missed. His final score was 120. How many times did he answer correctly?

**Step 1:** Let $r$ = number of right answers. Let $w$ = number of wrong answers. We know 2 sets of information that can be made into equations with 2 variables.
He earns +6 points for right answers and loses 12 points for wrong answers.

His wins and losses = 120

$$6r - 12w = 120$$
$$12w = r$$

12 times the number of wrong answers = the number of right answers.

**Step 2:** Substitute the value for $r$ ($12w$) into the first equation.

$$6(12w) - 12w = 120$$
$$w = 2$$

**Step 3:** Substitute the value for w back in the equation.

$$6r - 12(2) = 120$$
$$r = 24$$

**Answer:** He answered correctly 24 times.


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**Example 2:** Ms. Sudberry bought pencils and stickers for her first grade class on two different days. The pencils and stickers cost the same each time she went to the store. How much did she pay for each pencil?

| | Pencils | Stickers | Total Cost |
|---|---|---|---|
| Tuesday | 30 | 40 | \$47.50 |
| Saturday | 60 | 5 | \$20.00 |

**Step 1:** Set up two equations. Let the price of pencils equal $p$ and the price of stickers equal $s$.
The amount of the pencils times the price of pencils ($p$) plus the amount of the stickers times the price of stickers ($s$) equals the total amount paid for both pencils and stickers.

Equation 1: $30p + 40s = \$47.50$
Equation 2: $60p + 5s = \$20.00$

**Step 2:** Solve the equations by using one of the methods taught in this chapter. We will use the elimination method. First, multiply Equation 1 by $-2$, so $p$ will have the same coefficient in each equation but with opposite signs.

$-2\,(30p + 40s = \$47.50) = -60p - 80s = -\$95.00$

**Step 3:** Add the new Equation 1 to Equation 2.

$$\begin{array}{rrrrr} -60p & - & 80s & = & -\$95.00 \\ +\ 60p & + & 5s & = & \$20.00 \\ \hline & - & 75s & = & -\$75.00 \end{array}$$

The new equation is $-75s = -\$75.00$.

**Step 4:** Solve for $s$.

$-75s = -\$75.00$
$s = \$1.00$
Now we know the price of stickers, but the question asks for the price of each pencil.

**Step 5:** Substitute the value of $s$ into either equation and solve for $p$ to find the price of each pencil.

$30p + 40s = \$47.50$
$30p + 40(\$1.00) = \$47.50$
$30p + \$40.00 = \$47.50$
$30p = \$7.50$
$p = \$0.25$

**Answer:** The cost of each pencil is \$0.25.

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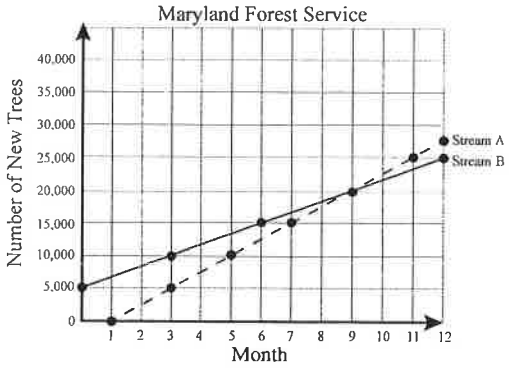
**Example 3:** Norman works for the Maryland Forest Service. He is in charge of planting trees in the "buffer zones" within 100 ft of two streams. The graph below shows the number of new trees planted along each stream by month:

At what point did the two streams have the same number of new trees? Up to that point, which stream had fewer new trees? After that point, which stream had fewer new trees?




**Answer:** The streams had the same number of new trees in month 9 (20,000 new trees).
Up to that point, Stream A had fewer new trees, and Stream B had more.
After that point, Stream B had fewer new trees, and Stream A had more.

**Use systems of equations to solve the following word problems. (DOK 3)**

1. The sum of two numbers is 126 and their difference is 42. What are the two numbers?

2. Prices for the movie are $4.00 for children and $8.00 for adults. The total amount of ticket sales is $1,176. There are 172 tickets sold. How many adults and children buy tickets?

3. A farmer sells a dozen eggs at the market for $2.00 and one of his bags of grain for $5.00. He has sold 5 times as many bags of grain as he has dozens of eggs. By the end of the day, he has made $243.00 worth of sales. How many bags of grain did he sell?

4. The choir sold boxes of candy and teddy bears near Valentine's Day to raise money. They sold 57 items altogether. Bears sold for $8.00 each and each box of candy sold for $6.00. They collected $380. How much of each item did they sell?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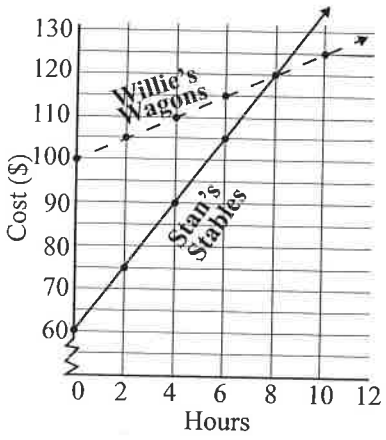
5. Timothy and Jesse went to purchase sports clothing they needed to play soccer. The table below shows what they bought and the amount they paid. What is the price of a soccer jersey and a pair of socks?

| | Soccer Jerseys | Tube Socks | Total Cost |
|---|---|---|---|
| Timothy | 4 | 7 | $78.30 |
| Jesse | 3 | 5 | $57.60 |

**Answer the following questions about the graph of system of equations. (DOK 2)**




6. Find the system of equations that represents the graph.

7. Willie's Wagons and Stan's stables have different prices when it comes to horseback riding lessons, but the price is the same at one point. How many hours would you sign up for horseback riding lessons for it to be the same price at Willie's Wagons and Stan's Stables?

8. Which place is more expensive for twelve or more hours of horseback riding lessons?

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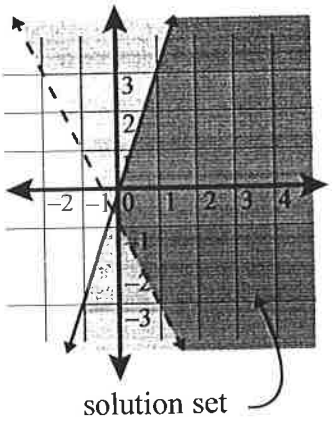
## 8.6 Graphing Systems of Inequalities (DOK 2)

Systems of inequalities are best solved graphically. Look at the following example.

**Example 1:** Sketch the solution set of the following system of inequalities:

$y > -2x - 1$ and $y \leq 3x$




Graph both inequalities on a Cartesian plane. Study the chapter on graphing inequalities if you need to review.

**Step 1:** Shade the portion of the graph that represents the solution set to each inequality just as you did in the chapter on graphing inequalities.

**Step 2:** Any shaded region that overlaps is the solution set of both inequalities.

**Graph the following systems of inequalities on your own graph paper. Shade and identify the solution set for both inequalities. (DOK 2)**

1. $9y \leq 6x + 18$
   $-4x - 4y \geq 8$

2. $x + y > -3$
   $-6x + 12y < -12$

3. $x - y \geq 2$
   $2y \leq -4x + 4$

4. $4x > 3y - 12$
   $-\frac{2}{3}x \geq y - 2$

5. $2y < -6x + 4$
   $y - 2x < -2$

6. $-2x + y > 2$
   $y \leq -x - 3$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## Chapter 8 Review

**For each pair of equations below, tell whether the graphs of the lines are coinciding, parallel, or intersecting. (DOK 2)**

1. $y = \frac{1}{2}x + 4$
   $y = \frac{1}{2}x - 3$

2. $3 - x = y$
   $2x + y = 1$

3. $y - \frac{1}{8}x = 2$
   $8y = 16 + x$

**Find the common solution for each of the following pairs of equations using the substitution method. (DOK 2)**

4. $2x - y = 3$
   $3x + 4y = -1$

5. $x + 2y = 1$
   $x + 3y = 1$

6. $-y = -2x + 7$
   $-x = -2y - 2$

7. $3x + 6y = 18$
   $4y - 2x = 12$

8. $2x - y = 10$
   $-\frac{1}{2}x = y$

9. $-2x + y = -6$
   $2x - 2y = 16$

**Find the point of intersection for each pair of equations using elimination. (DOK 2)**

10. $2x + y = 4$
    $3x - y = 6$

11. $x + 2y = 3$
    $x + 5y = 0$

12. $x + y = 1$
    $y = x + 7$

13. $2x + 4y = 5$
    $3x + 8y = 9$

14. $2x - 2y = 7$
    $3x - 5y = \frac{5}{2}$

15. $x - 3y = -2$
    $y = -\frac{1}{3}x + 4$

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**Graph the following systems of inequalities on your own graph paper. Shade and identify the solution set for both inequalities. (DOK 2)**

16. $y < 3x + 2$
    $y - \frac{1}{2}x > -1$

17. $4y \leq -3x - 12$
    $2y \leq x + 6$

**Answer the following questions about the description below. (DOK 3)**

Lucas is choosing between carpet cleaners for his home. Ability Best charges a $40 flat rate, then $2 per 100 square feet of carpet. Carpet Pro charges a $25 flat rate, then $3.50 per 100 square feet of carpet.

18. Find the system of equations represented by the description.

19. Use your own graph paper to graph the system of equations that you found in question above.

20. At what point do the two carpet cleaners charge the same amount for their services?

21. Lucas' house is 3,500 square feet. Which carpet cleaner should he use if he wants the better price?


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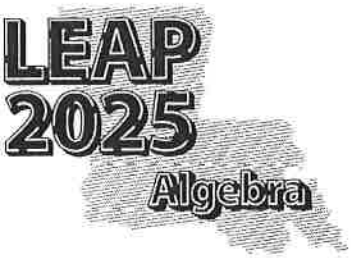
# Chapter 9
# Relations and Functions




Standards covered: F.IF.A.1, F.IF.A.2, F.IF.B.5, A.REI.D.11

## 9.1 Relations (DOK 2)

A **relation** is a set of ordered pairs. We call the set of the first members of each ordered pair the **domain** of the relation. We call the set of the second members of each ordered pair the **range**.

**Example 1:** State the domain and range of the following relation:

$\{(2, 4), (3, 7), (4, 9), (6, 11)\}$

**Answer:** Domain: $\{2, 3, 4, 6\}$ the first member of each ordered pair

Range: $\{4, 7, 9, 11\}$ the second member of each ordered pair

**State the domain and range for each relation. (DOK 2)**

1. $\{(2, 45), (33, 43), (98, 9), (43, 61), (67, 54)\}$

2. $\{(-7, -4), (-3, 16), (-4, 17), (-6, -8), (-8, 12)\}$

3. $\{(0, 9), (-8, 5), (3, 12), (-8, -3), (7, 18)\}$

4. $\{(58, 14), (44, 97), (74, 32), (6, 18), (63, 44)\}$

When given an equation in two variables, the domain is the set of $x$ values that satisfies the equation. The range is the set of $y$ values that satisfies the equation.

**Example 2:** Find the range of the relation $3x = y + 2$ for the domain $\{-1, 0, 1, 2\}$.

**Step 1:** Solve the equation for each value of $x$ given. Rearrange the equation to get y by itself. Doing so will give you $y = 3x - 2$. The result, the $y$ values, will be the range.

**Given:**

| $x$ | $y$ |
|---|---|
| $-1$ | |
| $0$ | |
| $1$ | |
| $2$ | |

**Solution:**

| $x$ | $y$ |
|---|---|
| $-1$ | $-5$ |
| $0$ | $-2$ |
| $1$ | $1$ |
| $2$ | $4$ |

**Answer:** The range is $\{-5, -2, 1, 4\}$.

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**Find the range of each relation for the given domain. (DOK 2)**

| | Relation | Domain | Range |
|---|---|---|---|
| 1. | $y = 5x$ | $\{1, 2, 3, 4\}$ | _____ |
| 2. | $y = -2x + 1$ | $\{0, 1, 3, 4\}$ | _____ |
| 3. | $y = 10x - 2$ | $\{-2, -1, 0, 1, 2\}$ | _____ |
| 4. | $y - x = 0$ | $\{1, 2, 3, 4\}$ | _____ |

**Follow the directions for the problem below. (DOK 3)**

5. Nasir and Ian decide to open a tutoring business. The two will evenly split the profit from the business. The office costs \$300 per month to rent. They charge \$30 an hour for their tutoring services.

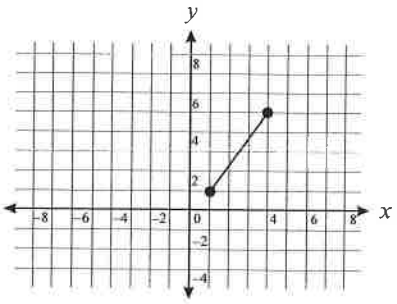
   A) Create a linear equation that models this situation.

   B) State the domain and range of the equation if they work 20 hours in January, 35 hours in February, 32 hours in March, and 44 hours in April.

## 9.2 Determining Domain and Range from Graphs (DOK 2)

The domain is all of the $x$ values that lie on the function in the graph, from the lowest $x$ value to the highest $x$ value. The range is all of the $y$ values that lie on the function in the graph, from the lowest $y$ value to the highest $y$ value.

**Example 1:** Find the domain and range of the graph.




**Step 1:** First find the lowest $x$ value depicted on the graph. In this case it is 1. Then find the highest $x$ value depicted on the graph. The highest value of $x$ on the graph is 4. The domain must contain all of the values between the lowest $x$ value and the highest $x$ value. The easiest way to write this is $1 \leq \text{Domain} \leq 4$ or $1 \leq x \leq 4$.

**Step 2:** Perform the same process for the range, but this time look at the lowest and highest $y$ values. The answer is $1 \leq \text{Range} \leq 5$ or $1 \leq y \leq 5$.



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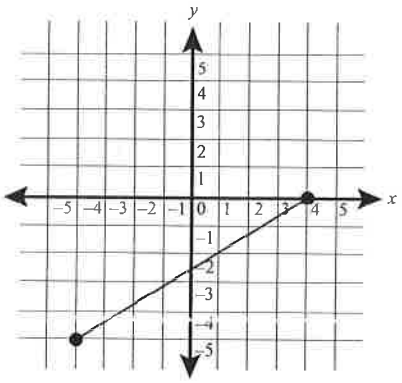
**Find the domain and range of each graph below. Write your answers in the line provided. (DOK 2)**

1.



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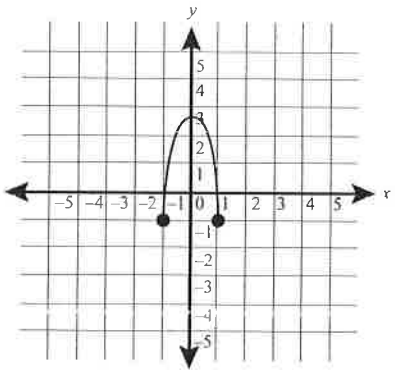

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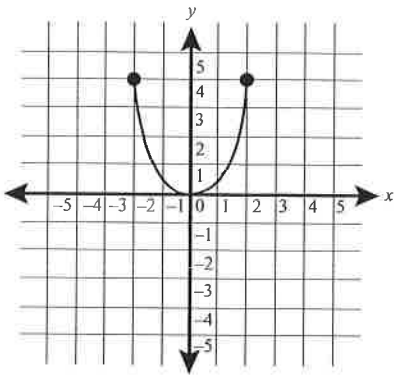

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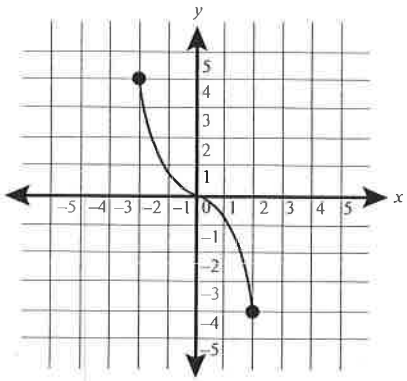

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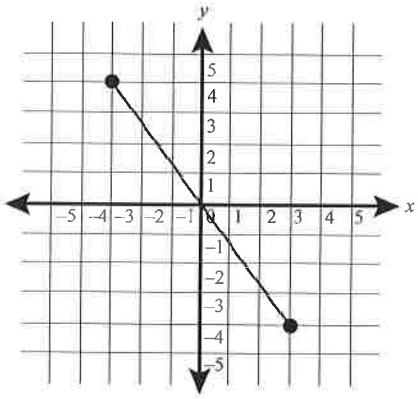

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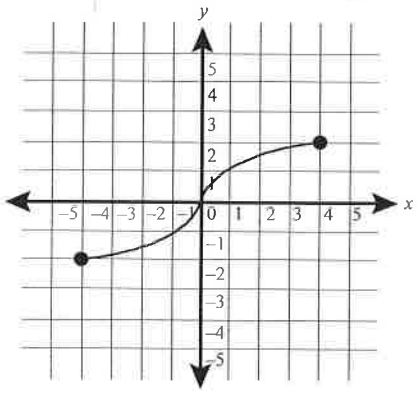

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## 9.3 Functions (DOK 1)

Some **relations** are also **functions**. A relation is a function if **for every element in the domain, there is exactly one element in the range**. In other words, for each value for $x$ there is only one unique value for $y$.

**Example 1:** $\{(2, 4), (2, 5), (3, 4)\}$ is **NOT** a function because in the first pair, 2 is paired with 4, and in the second pair, 2 is paired with 5. The 2 can be paired with only one number to be a function. In this example, the $x$ value of 2 has more than one value for $y$: 4 and 5.

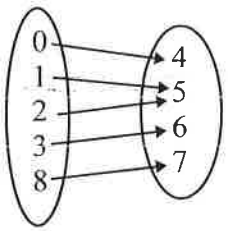
**Example 2:** $\{(1, 2), (3, 2), (5, 6)\}$ **IS** a function. Each first number is paired with only one second number. The 2 is repeated as a second number, but the relation remains a function.

**HINT:** In order to be a function, $x$ cannot repeat.

Functional relationships can be represented in a variety of ways. One way is to draw a picture that shows how ordered pairs are formed. These are called **function maps**. Each element in the domain of a relation is shown on the left, and each element of the range is shown on the right. The arrows create the ordered pairs by "mapping" each element of the domain onto the range. Remember, the relation is a function if each element in the domain is paired with exactly one element of the range.

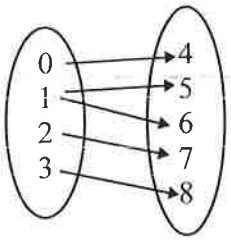
In the examples below, the arrows from different elements in the domain can map to the same element in the range, but the same element in the domain **CANNOT** map to different elements in the range.

Correct Mapping




**Function**

Incorrect Mapping




**Not a Function**

You could write out the ordered pairs from the maps and, if the same $x$-coordinate has more than one $y$-coordinate, then the relation is not a function. In the mapping above on the right, the ordered pairs mapped are (0, 4), (1, 5), (1, 6), (2, 7), and (3, 8). Because the $x$-coordinate $\{1\}$ has two different $y$-coordinates $\{5, 6\}$, we see once again that the relation is not a function.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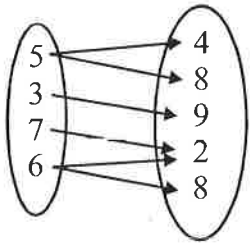
**Determine whether the ordered pairs of numbers below represent a function. Write "F" if it is a function. Write "NF" if it is not a function. (DOK 1)**

1. $\{(-1, 1), (-3, 3), (0, 0), (2, 2)\}$ _____
2. $\{(-3, 3), (0, 2), (1, 1), (2, 0)\}$ _____
3. $\{(-2, -5), (-2, -1), (-2, 1), (-2, 3)\}$ _____
4. $\{(0, 0), (1, 0), (2, 0), (3, 0)\}$ _____
5. $\{(8, -3), (-4, 4), (8, 0), (6, 2)\}$ _____
6. $\{(4, -3), (2, 0), (5, 3), (4, 1)\}$ _____

**Choose the correct answers below. (DOK 2, 3)**

7. Which set of ordered pairs completely expresses the relation shown?

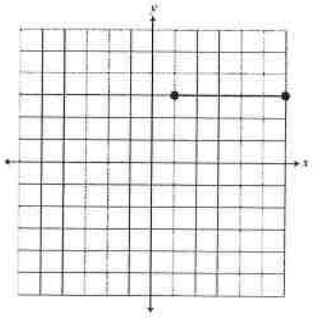



**A** (5, 4), (3, 9), (7, 2), (6, 8)
**B** (5, 8), (3, 9), (7, 2), (6, 2)
**C** (4, 5), (9, 3), (2, 7), (8, 6), (8, 5)
**D** (5, 4), (5, 8), (3, 9), (7, 2), (6, 2), (6, 8)

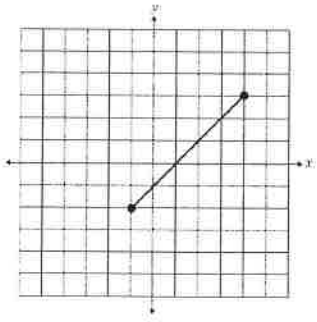
8. The relation mapped above
   **A** is a function because all of the $x$-coordinates are mapped to a $y$-coordinate.
   **B** is a function because all of the $y$-coordinates have at least one $x$-coordinate.
   **C** is not a function because all elements in the range are mapped.
   **D** is not a function because one element in the domain is mapped to two different elements in the range.

9. State the domain of graph #1 and the range of graph #2 below.

**Graph #1:**



**Graph #2:**



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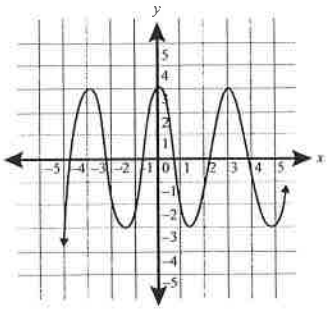
## 9.4 Recognizing Functions (DOK 1)

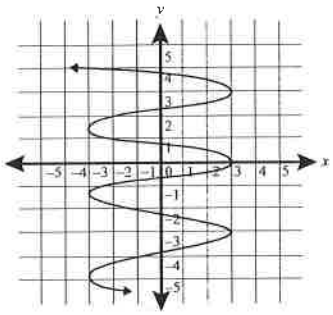
Recall that a relation is a function with only one $y$ value for every $x$ value. We can depict functions in many ways including through graphs.

**Example 1:**



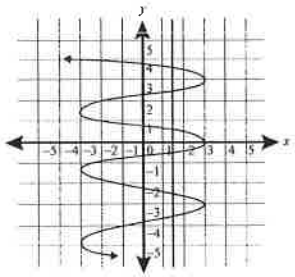
This graph **IS** a function because it has only one $y$ value for each value of $x$.

**Example 2:**



This graph is **NOT** a function because there is more than one $y$ value for each value of $x$.

**HINT:** One way to determine a function from a graph is to do a **vertical line test**. First, draw a vertical line that crosses over the whole graph. If the line crosses the graph more than one time, then it is not a function. If it only crosses it once, it is a function. Take the previous Example 2:



**Answer:** Since the vertical line passes over the graph more than once, it is not a function.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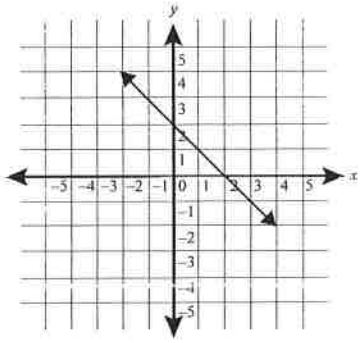
**Determine whether or not each of the following graphs is a function. If it is, write "function" on the line provided. If it is not a function, write "NOT a function" on the line provided. (DOK 1)**

1.



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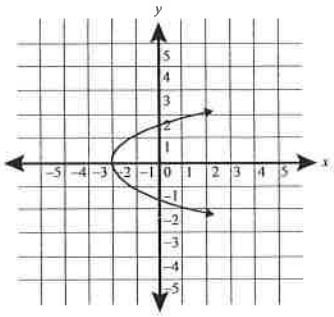
2.



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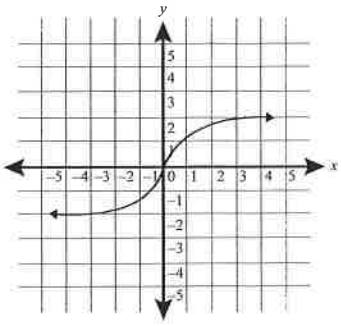
3.



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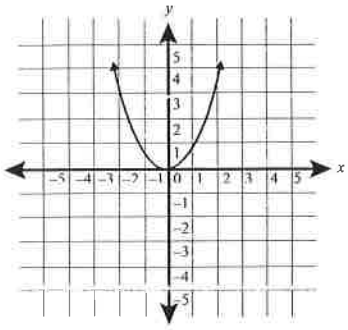
4.



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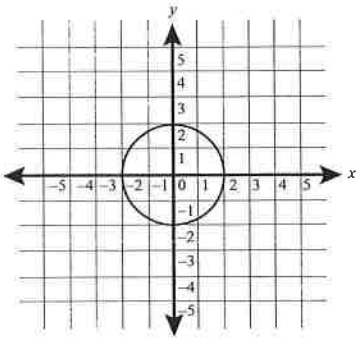
5.



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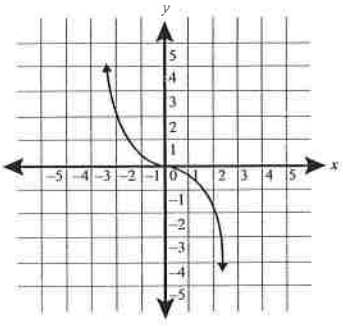
6.



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## 9.5 Function Notation (DOK 1)

Function notation is used to represent relations which are functions. Some commonly used letters to represent functions include $f$, $g$, $h$, $F$, $G$, and $H$. It's worth noting that the function notation $f(x)$ represents y in an equation in slope-intercept form. For example, saying $f(x) = 2x - 2$ is mathematically the same as $y = 2x - 2$.

**Example 1:** Given $g(x) = 4 - 2x^2$; find $g(2)$.

**Step 1:** Replace $x$ with 2 in the relation $4 - 2x^2$.

$g(2) = 4 - 2(2)^2$

**Step 2:** Solve $g(2)$.

$g(2) = 4 - 2(2)^2 = 4 - 2(4) = 4 - 8 = -4$

**Answer:** $g(2) = -4$

**Example 2:** Given the function $f(x) = 4x - 1$, find $f(x - 3)$.

**Step 1:** Substitute $x - 3$ into f $(x)$ for $x$.

$f(x - 3) = 4(x - 3) - 1$

**Step 2:** Simplify.

$f(x - 3) = 4(x - 3) - 1 = 4x - 12 - 1 = 4x - 13$

**Answer:** $f(x - 3) = 4x - 13$

**Find the solutions for each of the following. (DOK 1)**

1. For $F(x) = 2 + 3x^2$; find $F(3)$.
2. For $g(x) = -3x + 7$; find $g(-3)$.
3. For $f(x) = -5 + 4x$; find $f(7)$.
4. For $f(x) = 7 - 6x$; find $f(-4)$.
5. For $h(x) = 2x^2 + 10$; find $h(5)$.
6. For $F(x) = 7 - 5x$; find $F(2)$.

**Simplify. (DOK 2)**

7. For $f(x) = x + 3$, find $f(x - 1)$.
8. For $f(x) = x$, find $f(4x + 12)$.
9. For $h(x) = \frac{1}{2}x - 7$, find $h(6x)$.
10. For $f(x) = 3x - 2$, find $f(x + 1)$.



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## 9.6 Function Tables (DOK 2)

Functions can also use a variable such as $n$ to be the input of the function and $f(n)$, read as "$f$ of $n$," to represent the output of the function.

**Example 1:** Function rule: $3n + 4$

| $n$ | $f(n)$ |
|---|---|
| −1 | 1 |
| 0 | 4 |
| 1 | 7 |
| 2 | 10 |

**Fill in the tables for each function rule below. (DOK 2)**

1. rule: $2(n - 5)$

| $n$ | $f(n)$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |

2. rule: $3x(x - 4)$

| $x$ | $f(x)$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |

3. rule: $\frac{2 - n}{2}$

| $n$ | $f(n)$ |
|---|---|
| 0 | |
| 2 | |
| 4 | |
| 6 | |
| 8 | |

4. rule: $2x(x - 1)$

| $x$ | $f(x)$ |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |

5. rule: $\frac{1}{n + 3}$

| $n$ | $f(n)$ |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |

6. rule: $4x - x$

| $x$ | $f(x)$ |
|---|---|
| −2 | |
| −1 | |
| 0 | |
| 1 | |
| 2 | |

7. rule: $n(n + 2)$

| $n$ | $f(n)$ |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |

8. rule: $2x - 3$

| $x$ | $f(x)$ |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |

9. rule: $3 - 2n$

| $n$ | $f(n)$ |
|---|---|
| −2 | |
| −1 | |
| 0 | |
| 1 | |
| 2 | |

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## 9.7 Linear and Nonlinear Relations and Functions (DOK 1)

There are many different ways to determine whether or not a function is linear. The following examples will demonstrate how to tell.

**Example 1:** **Ordered Pairs** ⟶ In order for a set of ordered pairs to be linear, the slope between all points must be the same (constant).

**Step 1:** Find the slope between each of the points in the linear function below for verification.

Linear: $\{(0, 3), (1, 5), (2, 7)\}$

Slope $= \dfrac{y_1 - y_2}{x_1 - x_2}$

Slope between points 1 and 2: $\dfrac{5 - 3}{1 - 0} = 2$

Slope between points 2 and 3: $\dfrac{7 - 5}{2 - 1} = 2$

The slope is constant, so this is a linear function.

**Step 2:** Nonlinear: $\{(0, 1), (1, 0), (2, 1)\}$

Slope $= \dfrac{y_1 - y_2}{x_1 - x_2}$

Slope between points 1 and 2: $\dfrac{0 - 1}{1 - 0} = -1$

Slope between points 2 and 3: $\dfrac{1 - 0}{2 - 1} = 1$

The slope is different, so this is NOT a linear function.

**Example 2:** **Equations** ⟶ An equation can only be linear if it only has exponents of 1.

Linear: $y = 4x + 6$

Nonlinear: $y = 4x^2 + 6$

**Example 3:** **Tables** ⟶ Similar to ordered pairs, the slope between all points must be the same (constant).

Linear:

| $x$ | $y$ |
|---|---|
| −1 | −5 |
| 0 | 0 |
| 1 | 5 |

The slope is constant (5).

Nonlinear:

| $x$ | $y$ |
|---|---|
| −1 | 5 |
| 0 | 0 |
| 1 | 5 |

The slope is NOT constant.


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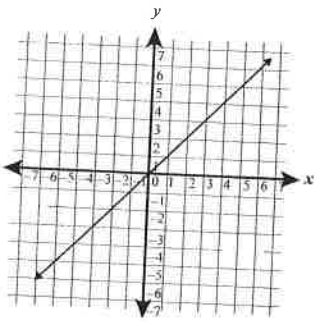
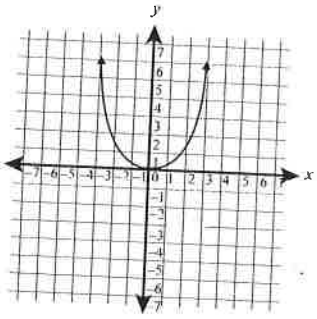
**Example 4:** **Graphs** → A graph easily determines whether or not a function is linear. If it is linear, it will be a straight line.

Linear:



Nonlinear:



**Tell whether the relation or function is linear or nonlinear. (DOK 1)**

1. $\{(4, 1), (4, 2), (4, 5)\}$

2. $y = 2x - 6$

3. $y = 2x^2 - 2x^2 + x + 1$

4.

| $x$ | $y$ |
|---|---|
| $-1$ | 6 |
| 0 | 0 |
| 1 | 6 |

**Follow the directions for each problem below. (DOK 3)**

5. Ms. Maxwell wrote four function tables on the board for her class, as shown below. Sort the tables into two groups: linear and nonlinear. Write an argument to present to your class justifying your answer.

| Table A | | | | |
|---|---|---|---|---|
| $x$ | $-4$ | 0 | 2 | 5 |
| $y$ | 7 | 3 | 1 | $-2$ |

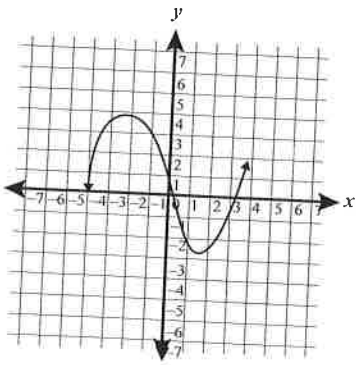
| Table B | | | | |
|---|---|---|---|---|
| $x$ | $-4$ | $-1$ | 2 | 5 |
| $y$ | 16 | 1 | 4 | 25 |

| Table C | | | | |
|---|---|---|---|---|
| $x$ | $-3$ | 0 | 2 | 4 |
| $y$ | $-27$ | 0 | 8 | 64 |

| Table D | | | | |
|---|---|---|---|---|
| $x$ | $-3$ | $-1$ | 3 | 5 |
| $y$ | $-14$ | $-8$ | 4 | 10 |

**Tell whether the relation or function is linear or nonlinear. (DOK 1)**

6.



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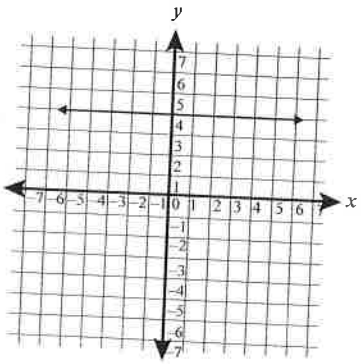

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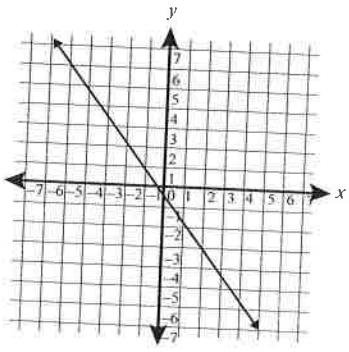

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## Chapter 9 Review

**Find the domain or range for each problem as directed. (DOK 2)**

1. What is the domain of the following relation? $\{(-1, 2), (2, 5), (4, 9), (6, 11)\}$

2. What is the range of the following relation? $\{(0, -2), (-1, -4), (-2, 6), (-3, -8)\}$

3. Find the range of the relation $y = 5x$ for the domain $\{0, 1, 2, 3, 4\}$.

4. Find the range of the relation $y = \frac{4+x}{3}$ for the domain $\{-7, -1, 2; 5, 8\}$.

**For each of the following relations given in questions 5–7, write F if it is a function and NF if it is not a function. (DOK 1)**

5. $\{(-1, 0), (0, 1), (1, 2), (2, 3)\}$
6. $\{(0, -1), (-1, -2), (-2, -3), (-3, -4)\}$
7. $\{(1, 7), (2, 5), (3, 6), (2, 4)\}$

**For questions 7–9, find the range of the following functions for the given value of the domain. (DOK 2)**

8. For $h(x) = x(x - 4)$; find $h(3)$.

9. For $f(x) = x + 11$; find $f(x + 1)$.

**Fill in the following function tables. (DOK 2)**

10. Rule: $\frac{(4-2n)}{2}$

| $n$ | $f(n)$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |

11. Rule: $6n - 3$

| $n$ | $f(n)$ |
|---|---|
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |

**Tell whether the relation or function is linear or nonlinear. (DOK 1)**

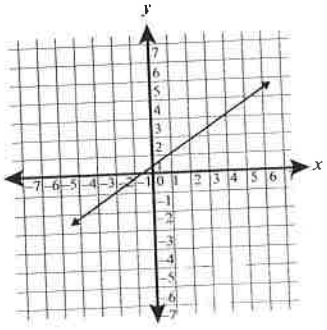
12. $\{(1, 2), (3, 7), (4, 5)\}$

13. $y = x^2 + x + 2$

14.

| $x$ | $y$ |
|---|---|
| −2 | 6 |
| −1 | 5 |
| 1 | 3 |
| 2 | 2 |

15.



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# Chapter 10
# Factoring

Standards covered: A.SSE.2, A.SSE.B.3

## 10.1 Greatest Common Factor of Polynomials (DOK 2)

In a multiplication problem, the numbers multiplied together are called **factors**. The answer to a multiplication problem is a called the **product**.

In the multiplication problem $5 \times 4 = 20$, the factors are 5 and 4, and 20 is the product.

If we reverse the problem, $20 = 5 \times 4$, we say we have **factored** 20 into $5 \times 4$.

In this chapter, we will factor polynomials.

**Example 1:** Find the greatest common factor of $2y^3 + 6y^2$.

**Step 1:** Look at the whole numbers. The greatest common factor of 2 and 6 is 2. Factor the 2 out of each term.

$2\,(y^3 + 3y^2)$

**Step 2:** Look at the remaining terms, $y^3 + 3y^2$. What are the common factors of each term?

$$\begin{array}{rcl} y^3 & = & y \times \boxed{y \times y} \\ 3y^2 & = & 3 \times \boxed{y \times y} \end{array} \longleftarrow \text{common factors} = y^2$$

**Step 3:** Now, factor $y^2$ out of each term: $2y^2\,(y + 3)$

**Check:** $2y^2\,(y + 3) = 2y^3 + 6y^2$

**Factor by finding the greatest common factor in each of the following. (DOK 2)**

1. $6x^4 + 18x^2$
2. $4b^5 + 12b^3$
3. $15a^3 - 25a^2$
4. $24b^3 + 16b^6$
5. $36y^4 + 9y^2$
6. $42x^3 + 49x$

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Factoring larger polynomials with 3 or 4 terms works the same way.

**Example 2:** $4x^5 + 16x^4 + 12x^3 + 8x^2$

**Step 1:** Find the greatest common factor of the whole numbers. 4 can be divided evenly into 4, 16, 12, and 8; therefore, 4 is the greatest common factor.

**Step 2:** Find the greatest common factor of the variables. $x^5$, $x^4$, $x^3$, and $x^2$ can be divided by $x^2$, the lowest power of $x$ in each term of the polynomial.

$4x^5 + 16x^4 + 12x^3 + 8x^2 = 4x^2 (x^3 + 4x^2 + 3x + 2)$

**Factor each of the following polynomials. (DOK 2)**

1. $18y^4 + 6y^3 + 24y^2$
2. $20x^4 + 16x^3 - 24x^2 + 28x$
3. $16x^5 + 20x^4 - 12x^3 + 24x^2$
4. $3n^5 + 9n^3 + 12n^2 + 15n$
5. $25p^5 - 10p^3 - 5p^2$
6. $y^3 + 3y^2 - 9y$

**Example 3:** Find the greatest common factor of $4a^3b^2 - 6a^2b^2 + 2a^4b^3$

**Step 1:** The greatest common factor of the whole numbers is 2.

$4a^3b^2 - 6a^2b^2 + 2a^4b^3 = 2(2a^3b^2 - 3a^2b^2 + a^4b^3)$

**Step 2:** Find the lowest power of each variable that is in each term. Factor them out of each term. The lowest power of $a$ is $a^2$. The lowest power of $b$ is $b^2$.

$4a^3b^2 - 6a^2b^2 + 2a^4b^3 = 2a^2b^2 (2a - 3 + a^2b)$

**Factor each of the following polynomials. (DOK 2)**

1. $12x^4y^3 + 18x^3y^4 - 24x^3y^3$
2. $20x^2y - 25x^3y^3$
3. $36c^4d + 42c^3d^3 + 24c^2d^2 - 18cd$
4. $21a^4b^3 + 27a^2b^3 + 15a^3b^2$
5. $5pq^3 - 2p^2q^2 - 9p^3q$
6. $33w^4y^2 - 9w^3y^2 + 24w^2y^2$


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## 10.2 Factoring By Grouping (DOK 2)

Not all polynomials have a common factor in each term. In this case, they may sometimes be factored by grouping.

**Example 1:** Factor $ab + 4a + 2b + 8$

**Step 1:** Factor an $a$ from the first two terms and a 2 from the last two terms.

$a(b + 4) + 2(b + 4)$

Now the polynomial has two terms, $a(b + 4)$ and $2(b + 4)$. Notice that $(b + 4)$ is a factor of each term.

**Step 2:** Factor out the common factor of each term:

$a(b + 4) + 2(b + 4) = (b + 4)(a + 2)$.

**Check:** Multiply using the FOIL method to check.

$(b + 4)(a + 2) = ab + 4a + 2b + 8$

**Factor the following polynomials by grouping. (DOK 2)**

1. $cd + 5c + 4d + 20$
2. $xy - 4x + 6y - 24$
3. $ab + 6a + 3b + 18$
4. $cd + 4c + 4d + 16$
5. $xy + 7x - 4y - 28$
6. $mn + 6m - 2n - 12$
7. $xy - 9x - 3y + 27$
8. $bc - 3b + 5c - 15$
9. $ab + a + 7b + 7$
10. $cd + 9c - d - 9$
11. $xy - 6x - 2y + 12$
12. $wz + 6z - 4w - 24$

## 10.3 Factoring Trinomials (DOK 2)

In the chapter on polynomials, you multiplied binomials (two terms) together, and the answer was a trinomial (three terms).

For example, $(x + 6)(x - 5) = x^2 + x - 30$.

Now, you need to practice factoring a trinomial into two binomials.

**Remember:**
Both signs (+) = +, +
Both signs (–) = +, –
1st sign (–), 2nd sign (+) = –, –
1st sign (+), 2nd sign (–) = +, –

**Example 1:** Factor $x^2 + 6x + 8$

**Step 1:** When the trinomial is in descending order as in the example above, you need to find a pair of numbers whose sum equals the number in the second term, while their product equals the third term.

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In this example, find the pair of numbers that has a sum of 6 and a product of 8.

_____ + _____ = 6 and _____ × _____ = 8

The pair of numbers that satisfy both equations is 4 and 2.

**Step 2:** Use the pair of numbers in the binomials.

The factors of $x^2 + 6x + 8$ are $(x + 4)$ $(x + 2)$.

**Check:** To check, use the FOIL method.

$(x + 4)(x + 2) = x^2 + 4x + 2x + 8 = x^2 + 6x + 8$

**Notice, when the second term and the third term of the trinomial are both positive, both numbers in the solution are positive.**

**Example 2:** Factor the following trinomial: $x^2 - x - 6$.

**Step 1:** Find the pair of numbers where the sum is $-1$ and the product is $-6$.

_____ + _____ = $-1$ and _____ × _____ = $-6$

The pair of numbers that satisfies both equations is 2 and $-3$.

The factors of $x^2 - x - 6$ are $(x + 2)(x - 3)$.

**Notice, if the second term and the third term are negative, one number in the solution pair is positive, and the other number is negative.**

**Example 3:** Factor the following trinomial: $x^2 - 7x + 12$.

**Step 1:** Find the pair of numbers where the sum is $-7$ and the product is 12.

_____ + _____ = $-7$ and _____ × _____ = 12

The pair of numbers that satisfies both equations is $-3$ and $-4$.

The factors of $x^2 - 7x + 12$ are $(x - 3)(x - 4)$.

**Notice, if the second term of a trinomial is negative and the third term is positive, both numbers in the solution are negative.**

Sometimes a trinomial has a greatest common factor which must be factored out first.

**Example 4:** Factor $4x^2 + 8x - 32$

**Step 1:** Begin by factoring out the greatest common factor, 4.

$4(x^2 + 2x - 8)$

**Step 2:** Factor by finding a pair of numbers whose sum is 2 and product is $-8$.
4 and $-2$ will work, so

$4(x^2 + 2x - 8) = 4(x + 4)(x - 2)$

**Check:** Multiply to check. $4(x + 4)(x - 2) = 4x^2 + 8x - 32$


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**Find the factors of the following trinomials. Be sure to factor out the greatest common factor first. (DOK 2)**

1. $k^2 - 4k + 3$
2. $b^2 - 4b - 5$
3. $c^2 - 5c - 14$
4. $a^2 - 9a + 20$
5. $w^2 + 13w + 36$
6. $y^2 + 14y + 49$
7. $3y^2 - 9y + 6$
8. $2a^2 + 2a - 12$
9. $4b^2 + 28b + 40$
10. A parallelogram has an area of $(x^2 - 7x + 12)$ and a height of $(x - 4)$. What is the expression which represents the base of the parallelogram? $A_{parallelogram} = bh$.

## 10.4 Factoring Trinomials with the "AC" Method (DOK 2)

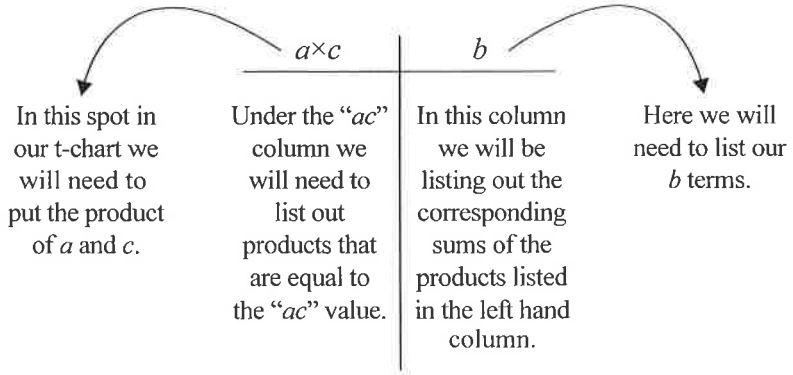
Sometimes when you factor trinomials, $ax^2 + bx + c$, $a > 1$ and cannot be factored out of the trinomial. When this happens, we can use a method known as the "AC" method. The "AC" method allows us to rewrite our trinomial into four separate terms so we can factor the expression by grouping instead of guess-and-check.

**Note:** This is not the only method that can be used to factor trinomials.

**Example 1:** Factor $24x^2 - 17x + 3$.

**Step 1:** First we must identify the coefficients of each term in our trinomial. In this case, $a = 24$, $b = -17$, and $c = 3$.

**Step 2:** Make a t-chart to organize your thoughts.

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So, in our particular example our t-chart would be as follow:

| $a \times c$ | $b$ |
|---|---|
| $24 \times 3 = 72$ | $-17$ |
| $-1 \times -72 = 72$ | $-1 - 72 = -73$ |
| $-2 \times -36 = 72$ | $-2 - 36 = -38$ |
| $-3 \times -24 = 72$ | $-3 - 24 = -27$ |
| $-4 \times -18 = 72$ | $-4 - 18 = -22$ |
| $-6 \times -12 = 72$ | $-6 - 12 = -18$ |
| $-8 \times -9 = 72$ | $-8 - 9 = -17$ |

**FYI:** Recall that the product of two negative numbers is positive, and the sum of two negative numbers is negative.

**Remember:** The commutative rule of multiplication and addition tells us we can multiply or add in any order.
For example: $-6 \times -12 = -12 \times -6$.
So, when you are listing out your products and you notice the numbers on the right are not on the left, you can be assured that you found all of your necessary products.

Notice how $-8 \times -9 = 72$ and $-8 - 9 = -17$. This tells us that $-8$ and $-9$ were the two numbers we were searching for.

**Step 3:** Now that we have found our two numbers it is time to break our trinomial up into 4 terms.

$24x^2 - 8x - 9x + 3$

Notice that if we were to combine like terms at this step it would take us right back to our original trinomial: $24x^2 - 17x + 3 = 24x^2 - 8x - 9x + 3$

**Step 4:** Now that we have four terms we can begin to factor by grouping.

Remember when grouping terms together to always keep the sign of the leading number. You can rearrange the terms in any order to help with factoring.

$\underline{24x^2 - 8x} \quad \underline{-9x + 3}$

Your goal here is that when you factor out the common term in each group the remaining terms inside each parenthesis will be identical. In this problem we see that after we factored out the common factor, each parenthesis, $(3x - 1)$, is **identical**. This tells us we are on track to factoring this problem correctly.

$8x(3x - 1) - 3(3x - 1)$

Our final step is to factor again, but this time we will be factoring out the common factor in each term. Notice that $8x(3x - 1)$ is one term and $-3(3x - 1)$ is our second term. Each term contains the factor $(3x - 1)$ which we will be factoring out. The result of what's left behind in each term is what gets placed in the second parenthesis.

$(8x - 3)(3x - 1)$

Therefore the factors of $24x^2 - 17x + 3$ are $(3x - 1)(8x - 3)$.


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**Factor the following trinomials. (DOK 2)**

1. $3y^2 + 14y + 8$
2. $5a^2 + 24a - 5$
3. $7b^2 + 30b + 8$
4. $2c^2 - 9c + 9$
5. $11a^2 + 35a + 6$
6. $5y^2 + 17y - 12$
7. $2y^2 - 17y + 36$
8. $11x^2 - 29x - 12$
9. $2x^2 - 3x - 20$
10. $3x^2 - 20x + 25$
11. $9y^2 - 5y - 4$

**Answer the question(s) for each problem below. (DOK 3)**

12. Dennis is building a rectangular sandbox for his granddaughter. The sandbox has an area of $(2x^2 + x - 1)$ and a length of $(x + 1)$. What is the polynomial expression which represents the width of the sandbox? What are the dimensions of the sandbox if $x = 3$ feet? $A_{rectangle} = lw$. [Hint: $(x + 1)$ (*what*?) $= 2x^2 + x - 1$].

## 10.5 Factoring Trinomials with Two Variables (DOK 2)

Some trinomials have two variables with exponents. You can still factor these trinomials.

**Example 1:** Factor $x^2 + 5xy + 6y^2$.

**Step 1:** Notice there is an $x^2$ in the first term and a $y^2$ in the last term. When you see two different terms that are squared, you know there has to be an $x$ and a $y$ in each factor:

$(x \quad y)(x \quad y)$

**Step 2:** Now think: What are two numbers whose sum is 5 and product is 6? You see that 3 and 2 will work. Put 3 and 2 in the factors:

$(x + 3y)(x + 2y)$

**Check:** Multiply to check. $(x + 3y)(x + 2y) = x^2 + 3xy + 2xy + 6y^2 = x^2 + 5xy + 6y^2$

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**Factor the following trinomials. (DOK 2)**

1. $a^2 + 6ab + 8b^2$
2. $c^2 - 2cd - 15d^2$
3. $a^2 - 5ab + 6b^2$
4. $x^2 + 5xy - 24y^2$
5. $a^2 - 4ab + 4b^2$
6. $g^2 - gh - 42h^2$
7. $x^2 + 12xy + 32y^2$
8. $a^2 - 2ab - 48b^2$
9. $x^2 + xy - 6y^2$

## 10.6 Factoring the Difference of Squares (DOK 2)

Let's give an example of a **perfect square**.

25 is a perfect square because $5 \times 5 = 25$.

49 is a perfect square because $7 \times 7 = 49$.

Any variable with an even exponent is a perfect square.

$y^2$ is a perfect square because $y \times y = y^2$.

$y^4$ is a perfect square because $y^2 \times y^2 = y^4$.

When two terms that are both perfect squares are subtracted, factoring those terms is easy. To factor the difference of perfect squares, you use the square root of each term, a plus sign in the first factor, and a minus sign in the second factor.

**Example 1:** Factor $4x^2 - 9$.

This example has two terms which are both perfect squares, and the terms are subtracted.

**Step 1:** $(2x \quad 3)\ (2x \quad 3)$

Find the square root of each term. Use the square roots in each of the factors.

**Step 2:** $(2x + 3)\ (2x - 3)$

Use a plus sign in one factor and a minus sign in the other factor.

**Check:** Multiply to check. $(2x + 3)\ (2x - 3) = 4x^2 - 6x + 6x - 9 = 4x^2 - 9$

Notice these two terms cancel out.



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**Example 2:** Factor $81y^4 - 1$.

**Step 1:** $(9y^2 + 1)(9y^2 - 1)$

Factor like the example above. Notice, the second factor is also the difference of two perfect squares.

**Step 2:** $(9y^2 + 1)(3y + 1)(3y - 1)$

Factor the second term further. **Note: You cannot factor the sum of two perfect squares.**

**Check:** Multiply in reverse to check your answer.

$(9y^2 + 1)(3y + 1)(3y - 1) = (9y^2 + 1)(9y^2 - 3y + 3y - 1) =$

$(9y^2 + 1)(9y^2 - 1) = 81y^4 + 9y^2 - 9y^2 - 1 = 81y^4 - 1$

**Factor the following differences of perfect squares. (DOK 2)**

1. $64x^2 - 49$
2. $4y^4 - 25$
3. $9a^2 - 16$
4. $c^4 - 36$
5. $25x^2 - 4$
6. $9x^4 - 64$
7. $16x^4 - 16$
8. $81x^4 - 16$
9. $x^6 - 9$

**Answer the question below. (DOK 3)**

10. A rectangle has an area of $4x^2 - 9$ and one side of the rectangle has a length of $(2x - 3)$. Find an expression that represents the missing side of the rectangle. $A_{rectangle} = lw$.

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## Chapter 10 Review

**Factor the following polynomials completely. (DOK 2)**

1. $8x - 18$
2. $15a^3 + 40$
3. $20y^6 - 12y^4$
4. $4y^2 - 36$
5. $3ax + 3ay + 4x + 4y$
6. $2bx + 2x - 2by - 2y$
7. $2b^2 - 2b - 12$
8. $7x^3 + 14x - 3x^2 - 6$
9. $12b^2 + 25b - 7$
10. $6y^2 + 30y + 36$
11. $16b^4 - 81d^4$
12. $12x^2 + 27x$
13. $2xy - 36 + 8y - 9x$
14. $2a^4 - 32$
15. $x^2 - y + xy - x$
16. $2b^3 - 24 + 16b - 3b^2$
17. $100y^4 - 49$
18. $g^2 + 7gh + 10h^2$

**Answer the question for each problem below. (DOK 3)**

19. A rectangular prism has a volume of $(25x^2 - 50x + 16)$ and a base area of $5x - 8$. What is the polynomial expression which represents the height of the prism? $V_{rectangular\ prism} = Bh$.

20. A parallelogram has an area of $(12x^2 + 52x + 16)$ and a height of $(2x + 8)$. What is the expression which represents the base of a parallelogram? $A_{parallelogram} = bh$.



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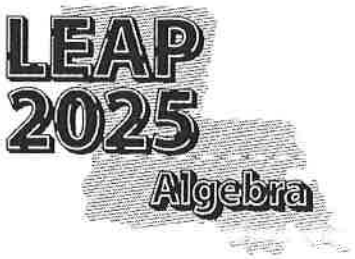
# Chapter 11
# Quadratic Equations and Functions




This chapter covers the following standards covered: A.SSE.3, A.CED.A.1, A.REI.A.4, F.IF.B.4, F.IF.B.6, F.IF.C.7a, F.IF.C.8a, F.BF.B.3, A-APR.B.3

## 11.1 Solving Quadratic Equations (DOK 2)

In the previous chapter, we factored polynomials such as $y^2 - 4y - 5$ into two factors:

$$y^2 - 4y - 5 = (y + 1)(y - 5)$$

In this chapter, we will talk about another type of equation. Any equation that can be put in the form $ax^2 + bx + c = 0$ is a quadratic equation if $a$, $b$, and $c$ are real numbers and $a \neq 0$. This is the standard form of a quadratic equation: $ax^2 + bx + c = 0$. To solve these equations, follow the steps below.

**Example 1:** Solve $y^2 - 4y - 5 = 0$.

**Step 1:** Factor the left side of the equation.

$$y^2 - 4y - 5 = 0$$

$$(y + 1)(y - 5) = 0$$

**Step 2:** If the product of these two factors equals zero, then the two factors individually must be equal to zero. Therefore, to solve, we set each factor equal to zero.

$$\begin{array}{r} (y+1) = 0 \\ -1 \quad -1 \\ \hline y = -1 \end{array} \qquad \begin{array}{r} (y-5) = 0 \\ +5 \quad +5 \\ \hline y = 5 \end{array}$$

The equation has two solutions: $y = -1$ and $y = 5$.

**Check:** To check, substitute each solution into the original equation.

When $y = -1$, the equation becomes:

$$(-1)^2 - (4)(-1) - 5 = 0$$
$$1 + 4 - 5 = 0$$
$$0 = 0$$

When $y = 5$, the equation becomes:

$$5^2 - (4)(5) - 5 = 0$$
$$25 - 20 - 5 = 0$$
$$0 = 0$$

Both solutions produce true statements.
The solution set for the equation is $\{-1, -5\}$.

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**Example 2:** Solve $2x^2 + 3x - 2 = 0$

$(2x - 1)(x + 2) = 0$

Set each factor equal to zero and solve:

$$\begin{array}{r} 2x-1 \;=0 \\ +1 \quad +1 \\ \hline \frac{2x}{2} = \frac{1}{2} \\ x = \frac{1}{2} \end{array} \qquad \begin{array}{r} x+2 \;=0 \\ -2 \quad -2 \\ \hline x = -2 \end{array}$$

The solution set is $\left\{\frac{1}{2}, -2\right\}$.

**Solve each of the following quadratic equations by factoring and setting each factor equal to zero. Check by substituting answers back in the original equation. (DOK 2)**

1. $b^2 - 9b + 14 = 0$
2. $y^2 + y - 20 = 0$
3. $x^2 - 3x - 28 = 0$
4. $b^2 + 7b + 12 = 0$
5. $7d^2 + 18d + 8 = 0$
6. $11x^2 - 31x - 6 = 0$
7. $5a^2 + 47a - 30 = 0$
8. $3x^2 - 10x + 8 = 0$
9. $8d^2 + 53d + 30 = 0$

## 11.2 Solving the Difference of Two Squares (DOK 2)

To solve the difference of two squares, first factor. Then set each factor equal to zero.

**Example 1:** $25x^2 - 36 = 0$

**Step 1:** Factor the left hand side of the equation.

$25x^2 - 36 = 0$

$(5x + 6)(5x - 6) = 0$

**Step 2:** Set each factor equal to zero and solve.

$$\begin{array}{r} 5x+6 \;=0 \\ -6 \quad -6 \\ \hline \frac{5x}{5} = \frac{6}{5} \\ x = -\frac{6}{5} \end{array} \qquad \begin{array}{r} 5x-6 \;=0 \\ +6 \quad +6 \\ \hline \frac{5x}{5} = \frac{6}{5} \\ x = \frac{6}{5} \end{array}$$



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**Check:** Substitute each solution in the equation to check.

for $x = -\frac{6}{5}$:

$25x^2 - 36 = 0$

$25\left(-\frac{6}{5}\right)^2 \left(-\frac{6}{5}\right) - 36 = 0$ ← Substitute $-\frac{6}{5}$ for $x$.

$\cancel{25}\left(\frac{36}{\cancel{25}}\right) - 36 = 0$ ← Cancel the 25.

$36 - 36 = 0$ ← A true statement. $x = -\frac{6}{5}$ is a solution.

for $x = \frac{6}{5}$:

$25x^2 - 36 = 0$

$25\left(\frac{6}{5}\right)^2 - 36 = 0$ ← Substitute $\frac{6}{5}$ for $x$.

$\cancel{25}\left(\frac{36}{\cancel{25}}\right) - 36 = 0$ ← Cancel the 25.

$36 - 36 = 0$ ← A true statement. $x = \frac{6}{5}$ is a solution.

The solution set is $\left\{-\frac{6}{5}, \frac{6}{5}\right\}$.

**Find the solution sets for the following. Show your work. (DOK 2)**

1. $25a^2 - 16 = 0$
2. $9x^2 - 64 = 0$
3. $d^2 - 25 = 0$
4. $81a^2 - 4 = 0$
5. $4c^2 - 49 = 0$
6. $49b^2 - 9 = 0$
7. $x^2 - 1 = 0$
8. $t^2 - 100 = 0$
9. $16k^2 - 81 = 0$

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## 11.3 Solving Perfect Squares (DOK 2)

When the square root of a constant, variable, or polynomial results in a constant, variable, or polynomial without irrational numbers, the expression is a perfect square. Some examples are 49, $x^2$, and $(x - 2)^2$.

**Example 1:** Solve the perfect square for $x$. $(x - 5)^2 = 0$

**Step 1:** Take the square root of both sides.

$\sqrt{(x-5)^2} = \sqrt{0}$

$(x - 5) = 0$

**Step 2:** Solve the equation.

$(x - 5) = 0$

$x - 5 + 5 = 0 + 5$

$x = 5$

**Example 2:** Solve the perfect square for $x$. $(x - 5)^2 = 64$

**Step 1:** Take the square root of both sides.

$\sqrt{(x-5)^2} = \sqrt{64}$

$(x - 5) = \pm 8$

$(x - 5) = 8$ and $(x - 5) = -8$

***Remember:** When taking the square root of a number, there are two answers.

$\sqrt{64} = 8$ and $-8$ or $\pm 8$

**Step 2:** Solve the two equations.

| | | |
|---|---|---|
| $(x - 5) = 8$ | and | $(x - 5) = -8$ |
| $x - 5 + 5 = 8 + 5$ | and | $x - 5 + 5 = -8 + 5$ |
| $x = 13$ | and | $x = -3$ |

**Solve the perfect square for $x$. (DOK 2)**

1. $(x + 1)^2 = 0$
2. $(x - 4)^2 = 0$
3. $(x + 8)^2 = 0$
4. $(x - 5)^2 = 16$
5. $(x + 9)^2 = 9$
6. $(x + 7)^2 = 36$
7. $(x + 2)^2 = 49$
8. $(x - 1)^2 = 4$
9. $(x + 8.9)^2 = 49$


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## 11.4 Rewriting the Equation by Completing the Square (DOK 2)

"Completing the square" is another way of factoring a quadratic equation. To complete the square, convert the equation into a perfect square.

**Example 1:** Rewrite $x^2 - 10x + 9 = 0$ by completing the square.

**Completing the square:**

$x^2 - 10x + 9 = 0$ is written in the form of $ax^2 + bx + c = 0$.

**Step 1:** Move $c$ to the other side and leave a space on both sides.

$x^2 - 10x$ _____ $= -9$ _____

The goal is to find a perfect square on the left side. In order to do this, we must divide "$b$" by 2, then square the result.

**Step 2:** In this example, "$b$" is –10. Divide it by 2 and square it.

$$\left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$$

**Step 3:** Place 25 in both blank spaces. We put it in both places because we follow the rules of algebra of what we do to the left side, we must do to the right side.

$x^2 - 10x + 25 = -9 + 25$

**Step 4:** Now make the left side a perfect square and combine terms on the right side.

$(x - 5)^2 = 16$

The equation is rewritten with perfect squares on both sides of the equation.

**Solving the perfect square:**

**Step 5:** Take the square root of both sides and set

$\sqrt{(x-5)^2} = \sqrt{16}$.

$(x - 5) = \pm 4 \rightarrow (x - 5) = 4$ and $(x - 5) = -4$

**Step 6:** Solve the two equations.

| | | |
|---|---|---|
| $(x - 5) = 4$ | and | $(x - 5) = -4$ |
| $x - 5 + 5 = 4 + 5$ | and | $x - 5 + 5 = -4 + 5$ |
| $x = 9$ | and | $x = 1$ |

**Solve for $x$ by completing the square. (DOK 2)**

1. $x^2 - 14x + 49 = 0$
2. $x^2 - 4x = 0$
3. $x^2 + 2x - 24 = 0$
4. $x^2 + 12x - 85 = 0$
5. $x^2 - 8x - 48 = 0$
6. $x^2 + 6x + 5 = 0$

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## 11.5 Completing the Square with a Leading Coefficient (DOK 2)

**Example:** Complete the square to rewrite the following equation: $2x^2 + 12x - 5 = 5$.

**Step 1:** Add 5 to both sides of the equation.

$2x^2 + 12x - 5 = 5 + 5$

$2x^2 + 12x = 10$

**Step 2:** Divide each term of the equation by the coefficient of $x^2$.

$$\frac{2x^2}{2} + \frac{12x}{2} = \frac{10}{2}$$

$$x^2 + 6x = 5$$

**Step 3:** Divide the coefficient of $x$ by 2 and square the result.

$(6 \div 2)^2 = (3)^2 = 9$

**Step 4:** Add the resulting value, 9, to both sides.

$x^2 + 6x + 9 = 5 + 9$

$x^2 + 6x + 9 = 14$

**Step 5:** Factor the $x^2 + 6x + 9$ into a perfect square.

$(x + 3)^2 = 14$

You have rewritten the equation.

**Complete the square to rewrite the equation.**

1. $2x^2 + 4x - 6 = 0$
2. $2x^2 - 16x = -30$
3. $x^2 - 6x - 7 = 0$
4. $3x^2 - 18x = -15$
5. $2x^2 - 16x = 0$
6. $x^2 - 2x - 15 = 0$
7. $x^2 - 2x - 48 = 0$
8. $2x^2 - 8x + 8 = 0$
9. $3x^2 + 6x = 24$
10. $x^2 + 4x - 21 = 0$
11. $x^2 + 6x - 16 = 0$
12. $2x^2 - 4x = 16$
13. $2x^2 - 12x = -10$
14. $2x^2 - 8x = -8$
15. $x^2 - 2x = 48$


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## 11.6 Using Quadratic Equations

You may be asked to use the quadratic formula to solve an algebra problem known as a **quadratic equation**. The equation should be in the form $ax^2 + bx + c = 0$.

**Example 1:** Using the quadratic formula, find $x$ in the following equation: $x^2 - 8x = -7$.

**Step 1:** Make sure the equation is set equal to 0.

$x^2 - 8x + 7 = -7 + 7$

$x^2 - 8x + 7 = 0$

The quadratic formula is $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

**Step 2:** In the formula, $a$ is the number $x^2$ is multiplied by, $b$ is the number $x$ is multiplied by and $c$ is the last term of the equation. For the equation in the example, $x^2 - 8x + 7$, $a = 1$, $b = -8$, and $c = 7$. When we look at the formula we notice a ± sign. This means that there will be two solutions to the equation, one when we use the plus sign and one when we use the minus sign.
Substituting the numbers from the problem into the formula, we have:

$$\frac{8 + \sqrt{8^2 - (4)(1)(7)}}{2(1)} = 7 \quad \text{or} \quad \frac{8 - \sqrt{8^2 - (4)(1)(7)}}{2(1)} = 1$$

**Answer:** The solutions are $\{7, 1\}$.

**For each of the following equations, use the quadratic formula to find two solutions. (DOK 2)**

1. $3x^2 - 9x - 30 = 0$

2. $4c^2 + 16c - 12 = 0$

3. $x^2 - 13x + 22 = 0$

4. $y^2 + 2y - 8 = 0$

5. $d^2 + 8d + 4 = -3$

6. $2a^2 - 16a - 14 = 0$

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## 11.7 Discriminant (DOK 2)

The **discriminant**, $D$, is determined from the coefficients of the quadratic equation $ax^2 + bx + c = 0$. The formula for the discriminant is shown below.

$$D = b^2 - 4ac$$

The discriminant illustrates how many real roots the quadratic equation has. Look at the table below:

| Discriminant | Roots |
|---|---|
| $D < 0$ | no real roots |
| $D = 0$ | one real roots |
| $D > 0$ | two real roots |

**Example 1:** Find the discriminant, $D$, of the equation $x^2 + 7x + 12 = 0$.

**Step 1:** Determine what $a$, $b$, and $c$ are in the quadratic equation $x^2 + 7x + 12 = 0$.
$a = 1$, $b = 7$, $c = 12$

**Step 2:** Substitute the values for $a$, $b$, and $c$ into the formula for the discriminant.
$D = b^2 - 4ac = 7^2 - 4(1)(12) = 49 - 48 = 1$

**Step 3:** The discriminant is 1. Looking at the chart above, we see that $1 > 0$, so there are two real roots of the quadratic equation $x^2 + 7x + 12 = 0$.

**Note:** The equation $x^2 + 7x + 12 = 0$ factors to $(x + 3)(x + 4) = 0$.

Solve for $x$:

$$\begin{array}{rcl} x + 3 & = & 0 \\ x & = & -3 \end{array} \qquad \begin{array}{rcl} x + 4 & = & 0 \\ x & = & -4 \end{array}$$

We see that the roots are $-4$ and $-3$. Therefore, there are two real roots for the quadratic equation $x^2 + 7x + 12 = 0$.

**Find the discriminant of the quadratic equations. (DOK 2)**

1. $x^2 - 9 = 0$

2. $2x^2 + 5x - 6 = 0$

3. $x^2 - 4x + 2 = 0$

4. $5x^2 - 6x + 21 = 0$

**First, find the discriminant. Then, determine how many real roots the equations have. (DOK 2)**

5. $3x^2 - x + 2 = 0$

6. $x^2 - 7x + 12 = 0$

7. $2x^2 - 20x + 50 = 0$

8. $x^2 - 10x + 24 = 0$



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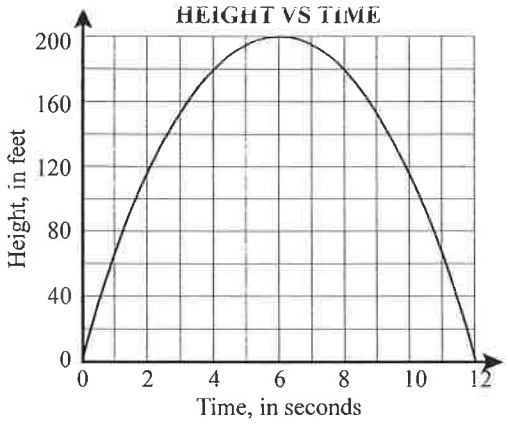
## 11.8 Real-World Functions (DOK 2)

Quadratic functions and their graphs are used to model many real-world situations. Some common examples are rockets that are launched, arrows shot into the air, golf balls hit into the air, and divers diving off of platforms. Instead of writing $y$ changing as a function of $x$, these variables are usually expressed as height changing as a function of time. The usefulness of these graphs is that, after a certain number of experiments, you can use the mathematical models to accurately predict future outcomes without performing any more experiments.

**Example 1:** An arrow was shot into the air. The graph shows the height, $h$, of its flight, $t$, seconds after it was shot.




**What can we say about this graph strictly from our knowledge of quadratic functions?** Time, $t$, in seconds is graphed along the $x$-axis. We know that the height of the arrow will depend on the time, so $t$ is the independent variable. The group of $t$ values will be the domain of the function as graphed along the $x$-axis, which describes the time that the arrow is in the air. Height, $h$, in feet, is graphed along the $y$-axis. Height, $h$, will be the dependent variable, and the group of all $h$ values will be the range of the function which describes the height of the flight path.

**What is the $x$-intercept of this graph?** Actually there are two places where this graph crosses the $x$-axis (or in this case, where $h = 0$). The first is at (0, 0). This is the point at which the arrow is on the ground prior to being shot. The second is at (12, 0). The arrow is back on the ground ($h = 0$) after flying for 12 seconds.

**What is the $y$-intercept?** This is the point where $x$ (or $t$ in this case) is equal to 0. At $t = 0$, $h = 0$. This means the $y$-intercept is prior to the arrow being shot at (0, 0).

**Does the function have a maximum or a minimum?** Yes. At $t = 6$ seconds, the arrow reaches a maximum height of 200 feet and begins descending. The minimum is when the arrow is at zero height and $t = 0$ and $t = 12$.

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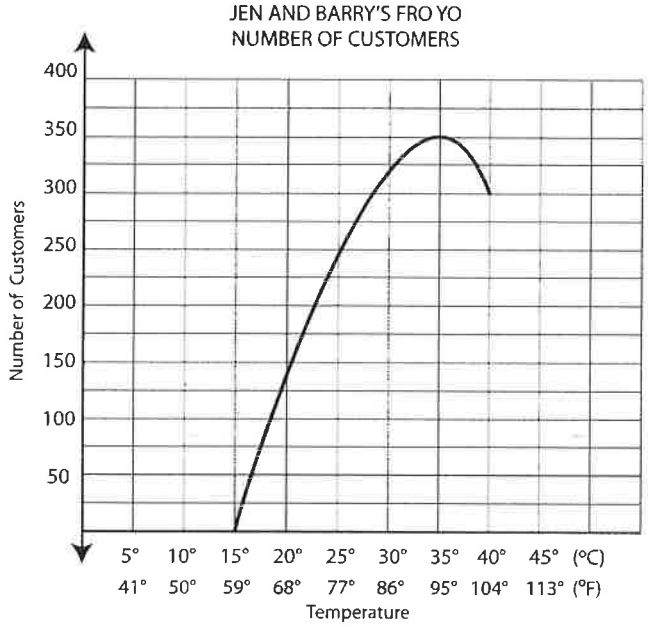
**Where does the arrow reach 100 feet?** Look closely. It reaches 100 feet the first time at $t = 2$ on the way up and again at $t = 10$ on the way down.

**What is the domain of the function?** The domain can be described as all the values of $t$ that make up the first coordinates of the ordered pairs of the graph. For this graph, $0 \leq t \leq 12$.

**What is the range of the function?** It is all the values of $h$ that make up the second coordinates of the ordered pairs of the graphs. In this example, the range is $0 \leq h \leq 200$.

Real-world graphs of function have various forms.

**Example 2:** Jen and Barry run a frozen yogurt stand in the spring and summer months. The graph below shows the number of customers they have on a weekday as a function of the high temperature. They do not open when the temperature is less than 15 °C (59 °F) (too cold) or over 40 °C (104 °F) (too hot to work in the stand).




What is the maximum number of customers that Jen and Barry have for any given temperature? At what temperature do they have this many customers? What is the range of this function? The domain?

The maximum point on the curve is (35,350), so the most customers that they will have is 350 customers. This maximum occurs at 35 °C (95 °F).

The $y$ values span from 0 customers to 350 customers, so the range is $0 \leq y \leq 350$, and the $x$ values span from 15 °C to 40 °C, so the domain is $15 \leq x \leq 40$.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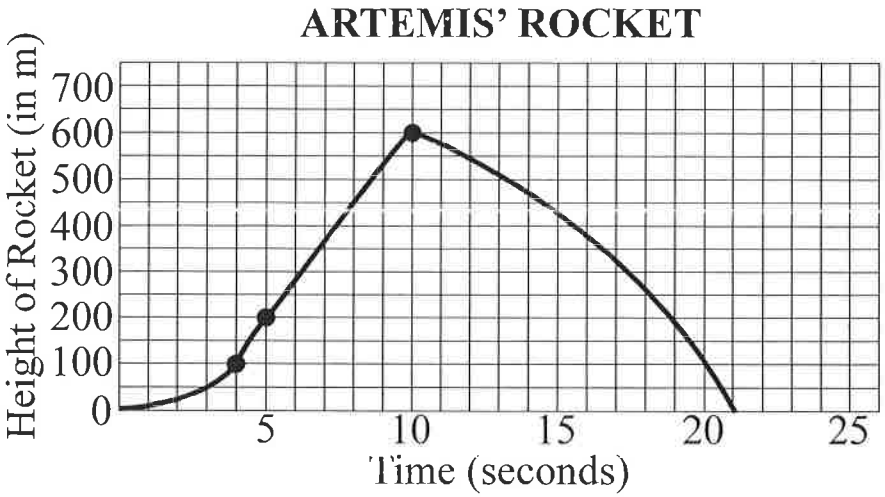
**Use the graphs to answer the questions. (DOK 2, 3)**

Artemis and his friends shoot off a miniature rocket. It accelerates for 5 seconds, ascends at a steady pace for 5 seconds, then runs out of fuel and falls to the ground. The graph below shows the flight of the rocket.




1. According to the graph, the rocket was 100 m high after 4 seconds. Approximately when did it reach 100 m again?

   **A** 10 seconds
   **C** 21 seconds
   **B** 20 seconds
   **D** 25 seconds

2. What was the rate of increase (slope) between 5 seconds and 10 seconds?

   **A** 40 m/sec
   **C** 80 m/sec
   **B** 75 m/sec
   **D** 100 m/sec

3. Find the rate of increase between 4 and 5 seconds. How does this compare to the rate of increase you found in question 2? Explain your answer on your own paper.

4. How long did the rocket's flight last? Write your answer as an ordered pair. This ordered pair represents what common characteristic of a graph? How do you know? Explain your answer on your own paper.

5. What is the maximum of this graph? What does the maximum represent in terms of the problem? Explain your answer on your own paper.

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## 11.9 Quadratic Equations Word Problems (DOK 3)

**Solve each problem for the indicated missing measurements. Explain your initial equation set up for each problem. (DOK 3)**

1. The hypotenuse of a certain right triangle can be expressed as $3x - 6$, and the legs of the triangle are $x$ and $2x + 6$. Find the lengths of the three sides of the triangle. (Hint: Use the Pythagorean Theorem to solve.)

2. Find the length and width of a rectangle that has an area of 28 $cm^2$ and a length that is three more than its width. $A_{rectangle} = lw$.

3. Find two consecutive positive integers such that 16 times the sum of the two integers minus 254 equals the product of the same two integers.

4. Terrance is having a rectangular swimming pool installed in his backyard. The plan for the pool shows an area of 108 $ft^2$ and the length is to be 3 feet more than the width. What is the length and width of Terrance's swimming pool?

## 11.10 Converting Quadratic Functions Between Standard and Vertex Form (DOK 2)

The **standard form** of a quadratic function is $f(x) = ax^2+bx+c$, where $a$, $b$, and $c$ are constants, and $a \neq 0$.

The **vertex form** of a quadratic function is $f(x) = a(x - h)^2 + k$, where $a$, $h$, and $k$ are constants, and $a \neq 0$. This equation is referred to as vertex form because the point $(h, k)$ is the vertex of the graph of the quadratic function.

**Example 1:** Convert the quadratic function $f(x) = 5(x + 2)^2 - 9$ from vertex form to standard form.

**Step:** To convert the function to standard form, simplify the function using order of operations.

$f(x) = 5(x + 2)^2 - 9$
$f(x) = 5(x + 2)(x + 2) - 9$
$f(x) = 5(x^2 + 4x + 4) - 9$
$f(x) = 5x^2 + 20x + 20 - 9$
$f(x) = 5x^2 + 20x + 11$


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**Answer:** The standard form of the function $f(x) = 5(x + 2)^2 - 9$ is $f(x) = 5x^2 + 20x + 11$.

**Example 2:** Convert the quadratic function $f(x) = 5x^2 - 40x + 67$ from standard form to vertex form.

**Step 1:** To convert the function to vertex form, you must first complete the square. The first step is to subtract the constant from both sides of the equation:

$f(x) = 5x^2 - 40x + 67$

$f(x) - 67 = 5x^2 - 40x + 67 - 67$

$f(x) - 67 = 5x^2 - 40x$

**Step 2:** Next, 5 should be factored from the right side of the equation:

$f(x) - 67 = 5x^2 - 40x = 5(x^2 - 8x)$

**Step 3:** Next, the square should be completed on the right side of the equation. To review completing, refer to Chapter 9. Divide the coefficient of $x$ by 2, then square the result. $(-8) \div 2 = -4$. $(-4)^2 = 16$. Add 16 to the binomial in the parenthesis.

$f(x) - 67 = 5(x^2 - 8x)$

$f(x) - 67 = 5(x^2 - 8x + 16)$

**Step 4:** Since 16 is added to the right side of the function within the parentheses, it is multiplied by 5. This means that we added 5 (16) = 80 to the right side of the function, not just 16. So we must add 80 to the left side of the function in order for the function to be balanced.

$f(x) - 67 + 80 = 5(x^2 - 8x + 16)$

**Step 5:** Next, simplify the function by factoring the trinomial on the right and combining like terms on the left.

$f(x) - 67 + 80 = 5(x^2 - 8x + 16)$

$f(x) + 13 = 5(x - 4)^2$

**Step 6:** Finally, 13 should be subtracted from both sides of the equation so that the function is in vertex form.

$f(x) + 13 = 5(x - 4)^2$

$f(x) + 13 - 13 = 5(x - 4)^2 - 13$

$f(x) = 5(x - 4)^2 - 13$ is the vertex form of the function.

Quadratic functions written in the form $y = a(x - h)^2 + k$ have a vertex at $(h, k)$.
Note the change in the sign of the $x$-value as you look at the equation and then locate the vertex.
The vertex is (4, –13),

**Convert each of the following quadratic functions from vertex form to standard form. (DOK 2)**

1. $f(x) = -3(x - 4)^2 + 7$

2. $f(x) = \frac{2}{3}(x + 1)^2 - 8$

3. $f(x) = 4(x - 7)^2 + 11$

4. $f(x) = 2(x - 5)^2 + 9$

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**Convert each of the following quadratic functions from standard form to vertex form. (DOK 2)**

5. $f(x) = -2x^2 - 24x - 75$

6. $f(x) = 5x^2 - 10x + 37$

7. $f(x) = 6x^2 + 72x + 228$

8. $f(x) = 7x^2 + 28x + 19$

## 11.11 Characteristics of Quadratic Functions (DOK 2)

Every quadratic function has various characteristics.

The **domain** of the quadratic function $f(x)$ is all possible values of $x$ for the function, the **range** is all possible values of $f(x)$.

The **zeros** or **roots** are the values of $x$ that produce a value of 0 for $f(x)$, and the **extrema** is the maximum or minimum value of $f(x)$. The zeros also determine where the $x$-intercepts exist on the function's graph.

The graph of every quadratic function, which is a parabola, has certain properties as well.

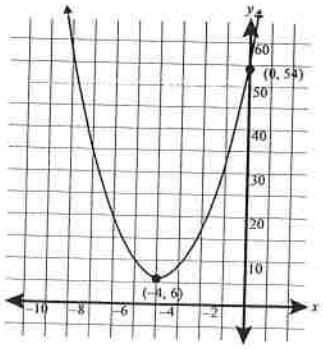
The **axis of symmetry** of the graph of the function $f(x)$ is a line down the middle of the graph that passes through the vertex and divides the parabola into two equal parts.

The **vertex** of the graph is the point of intersection between the axis of symmetry and the parabola. It is the highest or lowest point in the center of a parabola.

The **intercepts** of the graph are the points where the parabola crosses the $x$- and $y$-axis.

The **intervals of increase and decrease** of the graph of the function $f(x)$ are the regions in which the parabola is going up or going down.

**Example 1:** The graph of the function $f(x) = 3(x + 4)^2 + 6$ is shown. List the function's domain, range, zeros, and extrema, and the graph's axis of symmetry, vertex, intercepts, and intervals of increase/decrease.




**Domain:** all real numbers

**Range:** $f(x) \geq 6$. The range of the function is $6 \leq y < \infty$.

**Zeros:** no zeros (The function does not cross $x$-axis, so $f(x)$ never equals zero.)



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**Extrema:** The function has a minimum at 6.

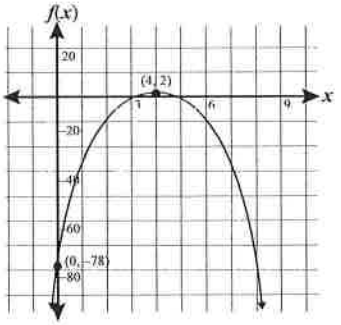
**Symmetry:** Since the graph of the function can be reflected over the line $x = -4$ and produce the same graph, its axis of symmetry is $x = -4$.

**Vertex:** The graph's vertex is $(-4, 6)$. This can be found from the vertex form.

**Intercepts:** The graph does not have any $x$-intercepts. The function has a $y$-intercept at $(0, 54)$.

**Intervals:** The graph decreases from $-\infty$ to $-4$ and increases from $-4$ to $\infty$.

**Example 2:** The graph of the function $f(x) = -5(x - 4)^2 + 2$ is shown. List the function's domain, range, zeros, and extrema, and the graph's axis of symmetry, vertex, intercepts, and intervals of increase/decrease.




**Domain:** all real numbers

**Range:** The range of the function is all values less than or equal to 2: $-\infty < y \leq 2$.

**Zeros:** The zeros, which can be determined by plugging 0 in for $f(x)$ and solving for $x$, are 4.63 and 3.36.

**Extrema:** Where $x = 4$, $f(x) = 2$. For all other values of $x$, $f(x) < 2$. Therefore, the function has a maximum value of 2, the extrema.

**Symmetry:** The graph of the function can be reflected on the line $x = 4$ and produce the same graph, its axis of symmetry is $x = 4$.

**Vertex:** The axis of symmetry intersects the parabola at the point (4, 2), the graph's vertex is (4, 2).

**Intercepts:** The graph has $x$-intercepts at $x = 3.36$ and $x = 4.63$. Plugging in zero for $x$ and solving for $f(x)$ shows that the function has a $y$-intercept at $f(x) = -78$.

**Intervals:** The graph decreases from 4 to $\infty$ and increases from $-\infty$ to 4.

**Example 3:** Determine the equation of the quadratic function whose graph has a vertex of (5, 8) and a $y$-intercept of $(0, -67)$.

**Step 1:** Since the quadratic function has a graph whose vertex is (5, 8), the equation of the function must be $f(x) = a(x - 5)^2 + 8$, where $a$ is a constant. Also, since the $y$-intercept of the graph is $f(x) = -67$, when 0 is plugged in for $x$, the value of $f(x)$ must equal $-67$. For this reason, $a$ can be solved for as follows:

$-67 = a(0 - 5)^2 + 8$

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$-67 = a(5)^2 + 8$

$-67 = 25a + 8$

$-67 - 8 = 25a + 8 - 8$

$-75 = 25a$

$a = -3$

**Answer:** Therefore, the equation of the quadratic function must be $f(x) = -3(x-5)^2 + 8$.

**For each of the following quadratic functions, list the function's domain, range, zeros, and extrema, and for the graph of the function list the axis of symmetry, vertex, intercepts, and intervals of increase and decrease. (DOK 2)**

1. $f(x) = \frac{1}{4}(x + 9)^2 + 1$

2. $f(x) = 9(x - 1)^2 + 10$

3. $f(x) = -2(x + 8)^2 + 5$

4. $f(x) = 5(x - 3)^2 - 11$

**Determine the equation of the quadratic function that has each of the following vertices and *y*-intercepts. (DOK 2)**

5. Vertex: (5, 2); *y*-intercept: (0, 152)

6. Vertex: (−4, −11); *y*-intercept: (0, −9)

**Use the general function of a vertex form expressed as $y = a(x - h)^2 + k$ to answer the following questions. (DOK 3)**

7. What effect does the positive or negative value of $a$ have on the graph of the quadratic function?

8. The ($h$) value in vertex form can be difficult for students because the positive or negative value does not match the direction shift we typically associate with it. For example, $y = (x+3)^2 - 6$ indicates the graph is being shifted towards the left. Typically, +3 is thought of as shifting to the right. Write a letter to next year's Algebra I students explaining why this true. Offer any tips, suggestions, or advice you can think of to help students remember this concept.



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9. Cityville Electric Company charges their customers a maintenance fee of \$3.99 per month in addition to their usage fee. The usage fee is calculated at \$0.09 for each of the first 600 kilowatts used and \$0.18 each for any additional kilowatts used.

   A) If the Roger's family received a bill in the amount of \$84.81, how many kilowatts did they use last month?

   B) What is the domain and range of this problem?

   C) What are the intercepts, if any? Interpret their meaning in terms of the problem.

   D) Does this function have a maximum or minimum? If so, define them, and explain how you know. If not, explain why not.

## 11.12 Rate of Change of Quadratic Functions (DOK 2)

The average rate of change for a function is the change in $f(x)$ divided by the change in $x$. In linear equations, the rate of change (slope) is constant. In quadratic functions, the rate of change is not constant. For any point on a parabola, the rate of change is different. The average rate of change for $f(x)$ from one value of $x$, $x_1$, to another, $x_2$, can be calculated:

$$\frac{f(x_1) - f(x_2)}{x_1 - x_2}$$

By choosing a value of $x_2$ that is close to $x_1$ we can approximate the rate of change of $f$ at $x_1$. Remember, you must keep the order of the terms in the numerator and the denominator consistent. A positive rate of change will have a graph that "goes up." A negative rate of change will have a graph that "goes down." A rate of change of 0 indicates a horizontal line. A very large rate of change indicates a nearly vertical line.

**Example 1:** What is the average rate of change for $f(x) = -2(x + 2)^2 + 2$ from $x = -2$ to $x = 2$?

**Step 1:** Find $f(-2)$ and $f(2)$ by plugging in $x = -2$ and $x = 2$ into the equation.
$f(-2) = -2(-2 + 2)^2 + 2 = 2$
$f(2) = -2(2 + 2)^2 + 2 = -30$

**Step 2:** Find the difference between $f(2)$ and $f(-2)$. Plug in the values for $x$.
$f(2) - f(-2) = -30 - 2 = -32$

**Step 3:** Find the difference between 2 and $-2$.
$2 - (-2) = 4$

**Step 4:** Divide the change in $f(x)$ by the change in $x$. $-32 \div 4 = -8$

**Answer:** The average rate of change of the function from $x = -2$ to $x = 2$ is $-8$.

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**Find average rate of change of each of the following quadratic functions from $x = 2$ to $x = 5$. (DOK 2)**

1. $f(x) = 7(x - 1)^2 - 3$
2. $f(x) = x^2 - 6x + 19$
3. $f(x) = 4x^2 - 7x - 2$
4. $f(x) = -2(x + 9)^2 + 4$

## Chapter 11 Review

**Factor and solve each of the following quadratic equations. (DOK 2)**

1. $16b^2 - 25 = 0$
2. $100x^2 - 49 = 0$
3. $81y^2 = 9$
4. $y^2 - 7y + 8 = 16$
5. $4x^2 + 19x - 5 = 0$
6. $-6x^2 + 7x - 2 = 0$

**Using the quadratic formula, find both solutions for the variable. (DOK 2)**

7. $x^2 + 10x - 11 = 0$
8. $y^2 - 14y + 40 = 0$
9. $-3x^2 - 2x - 2 = 0$
10. $4x^2 + x + 5 = 0$

**Solve the problems as indicated below. (DOK 3)**

11. Find two consecutive positive odd integers such that twice the sum of the largest number plus 77 is equal to the product of both numbers.


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12. Amber has a mini dry erase board that has an area of 65 $in^2$. The length of the dry erase board is two less than triple the width. What is the length and the width?

**Convert each of the following quadratic functions from vertex form to standard form. (DOK 2)**

13. $f(x) = 16(x - 1)^2 + 2$

14. $f(x) = \frac{4}{5}(x + 5)^2 - 9$

**Convert each of the following quadratic functions from standard form to vertex form. (DOK 2)**

15. $f(x) = \frac{1}{2}x^2 + 9x + \frac{61}{2}$

16. $f(x) = -3x^2 + 48x - 222$

**For each of the following quadratic functions, list the function's domain, range, zeros, and extrema, and for the graph of the function list the axis of symmetry, vertex, intercepts, and intervals of increase and decrease. (DOK 2)**

17. $f(x) = 14(x - 3)^2 + 1$

18. $f(x) = -8(x + 5)^2 + 28$

**Find the average rate of change of each of the following quadratic functions from $x = 1$ to $x = 5$. (DOK2)**

19. $f(x) = 5(x - 12)^2 - 2$

20. $f(x) = \frac{1}{5}x^2 - 4x + 13$

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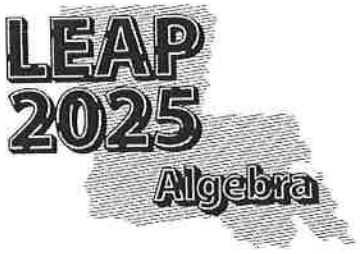
# Chapter 12
# Exponential Functions




Standards covered: A.CED.A.1, F.IF.B.4, F.IF.B.6, F.IF.C.7e, F.IF.C.8b, F.BF.B.3, F.LE.A.1, F.LE.A.2, F.LE.B.5

## 12.1 Characteristics of Exponential Functions (DOK 3)

An **exponential function** is a function in the form $f(x) = a^x$, where $a$ is a constant. Every exponential function has various characteristics, such as its domain, range, and zeros, and the graph of every exponential function has certain properties as well, such as the graph's asymptotes, intercepts, intervals of increase/decrease, and end behavior.

The **domain** of the exponential function $f(x)$ is all real numbers.

The **range** is all possible values of $f(x)$. In the basic function $f(x) = a^x$, the range is $(0, \infty)$.

The **zeros** are the values of $x$ that produce a value of 0 for $f(x)$.

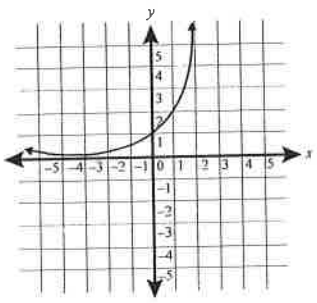
The **asymptotes** of the graph of the function $f(x)$ are the lines that the graph approaches but never touches. In the basic function $f(x) = a^x$, the horizontal asymptote is $y = 0$.

The **intercepts** of the graph are the points where the graph crosses the $x$- and $y$-axes.

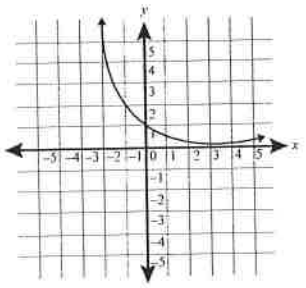
The **intervals of increase or decrease** of the graph of the function $f(x)$ are the regions in which the graph is going up or going down.

The **end behavior** of the graph is what happens to the graph as $x$ approaches negative infinity or infinity.

Increasing if $a > 1$



Decreasing if $a < 1$





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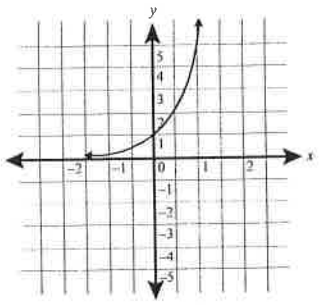
**Example 1:** The graph of the function $f(x) = 5^x$ is shown. List the function's domain, range, and zeros, and the graph's asymptotes, intercepts, intervals of increase/decrease, and end behavior.



**Domain**: all real numbers
**Range**: Since $f(x)$ is always positive, the range of the function is $(0, \infty)$.
**Zeros**: The function doesn't have any zeros because it never crosses the $x$-axis.
**Asymptotes**: The graph of the function approaches the $x$-axis but never touches it, so the graph has an asymptote at $y = 0$.
**Intercepts**: The $y$-intercept is (0, 1), and the graph doesn't have any $x$-intercepts.
**Intervals**: As $x$ goes from negative infinity to infinity, the graph of the function is always going up, so it has an interval of increase of $(-\infty, \infty)$, and there is no interval of decrease.
**End Behavior**: As $x$ approaches negative infinity, $f(x)$ approaches 0, and as $x$ approaches infinity, $f(x)$ approaches infinity.

**Example 2:** Give an example of an exponential function whose graph passes through the point (6, 1).
It is already known that the graph of an exponential function in the form $f(x) = a^x$ passes through the point (0, 1) because if 0 is plugged in for $x$, the function becomes $f(x) = a^0 = 1$.
Therefore, for an exponential function's graph to pass through the point (6, 1), if 6 is plugged into the function for $x$, $f(x)$ would have to equal 1.
An example of an exponential function that meets this condition is one in the form $f(x) = a^{x-6}$. If 6 is plugged into the function for $x$, the function becomes $f(x) = a^{6-6} = a^0 = 1$.
Since $a$ can equal any real number, one example of an exponential function whose graph passes through the point (6, 1) is $f(x) = 8^{x-6}$.

**For each of the following exponential functions, list the function's domain, range, and zeros, and for the graph of the function list the asymptotes, intercepts, intervals of increase/decrease, and end behavior. (DOK 3)**

1. $f(x) = -3^{-x}$
2. $f(x) = \left(\frac{1}{3}\right)^{-x}$
3. $f(x) = \left(\frac{1}{5}\right)^{x}$
4. $f(x) = 6^x$

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**Give an example of an exponential function that meets each of the following criteria. (DOK 3)**

5. The graph of the function passes through the point (–3, 1).

6. The function has a zero at $x = \frac{1}{2}$.

## 12.2 Rate of Change of Exponential Functions (DOK 2)

The average rate of change of the exponential function $f(x)$ is the change in the function over a given interval of $x$. In exponential functions, the rate of change is not constant. The average rate of change for $f(x)$ from one value of $x$, $x_1$, to another, $x_2$, can be calculated:

$$\frac{f(x_1) - f(x_2)}{x_1 - x_2}$$

**Example 1:** Calculate the average rate of change of the exponential function $f(x) = 2^x$ from $x = 1$ to $x = 3$.

**Step 1:** Find $f(1)$ and $f(3)$ by plugging in $x = 1$ and $x = 3$ into the equation.

$f(1) = 21 = 2$

$f(3) = 23 = 8$

**Step 2:** Find the average rate of change.

$$\frac{f(1) - f(3)}{1 - 3}$$

So, $\frac{2-8}{1-3} = \frac{-6}{-2} = 3$

**Answer:** The average rate of change of $f(x)$ from $x = 1$ to $x = 3$ is 3.

**Find the average rate of change of each of the following exponential functions from $x = 1$ to $x = 4$. (DOK 2)**

1. $f(x) = 5^x$

2. $f(x) = \left(\frac{1}{2}\right)^{-x} - 6$

3. $f(x) = 3^{x+1}$

4. $f(x) = -3^x + 8$



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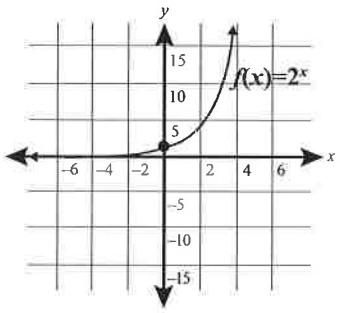
## 12.3 Transforming Graphs of Exponential Functions (DOK 2)

An exponential function $f(x) = a^x$ may be as complicated as $f(x) = ba^{c(x-d)} + k$.
An exponential function in this form may be graphed by transforming the graph of $f(x) = a^x$. Six types of transformations can occur:

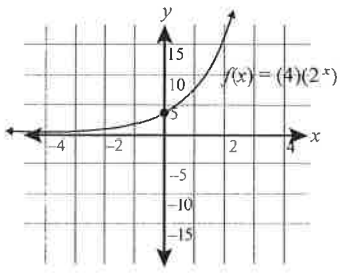
1. If $b \neq 1$, then vertical stretching/shrinking of the graph occurs. Vertically stretch the graph if $|b| > 1$, and compress the graph if $|b| < 1$.
2. If $b$ is negative, reflect the graph across the $x$-axis.
3. If $k \neq 0$, then a translation vertically will occur. Move the graph $k$ units up if $k$ is positive and $k$ units down if $k$ is negative.
4. If $c \neq 1$, then horizontal stretching/shrinking of the graph occurs. Horizontally stretch the graph if $|c| < 1$, and compress the graph if $|c| > 1$.
5. If $c$ is negative, reflect the graph across the $y$-axis.
6. If $d \neq 0$, then a horizontal translation will occur. Move the graph $d$ units to the right if $d$ is positive and $d$ units to the left if $d$ is negative.

**Note:** It does not matter which type of transformation happens first.

**Example 1:** Graph the exponential function $f(x) = (-4)(2^x) - 5$ as a transformation of the function $f(x) = 2^x$.
The graph of the function $f(x) = 2^x$ is graphed below.




**Step 1:** To transform the graph of the function $f(x) = 2^x$ into the graph of the function $f(x) = (-4)(2^x) - 5$, the first transformation that will be applied is a vertical stretch by a factor of 4. When this is done, the function becomes $f(x) = (4)(2^x)$.

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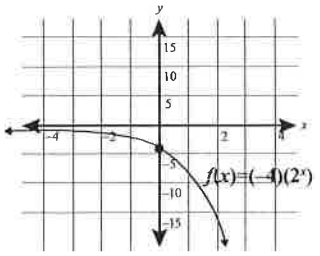
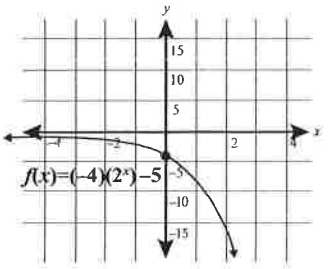
**Step 2:** Next, a reflection across the $x$-axis will be applied. When this is done, the function becomes $f(x) = (-4)(2^x)$.




**Step 3:** Finally, a translation of 5 units down will be applied. When this is done, the function becomes $f(x) = (-4)(2^x) - 5$.




**Use your own paper to graph each exponential functions as transformations of $f(x) = 4^x$. (DOK 2)**

1. $f(x) = (-3)(4^x)$
2. $f(x) = (7)(4^x) - 3$
3. $f(x) = -4^x + 6$
4. $f(x) = (-5)(4^x) + 4$

**On your own paper, graph each exponential functions as transformations of $f(x) = \left(\frac{1}{2}\right)^x$. (DOK 2, 3)**

5. $f(x) = \left(\frac{1}{2}\right)^{8x}$
6. $f(x) = \left(\frac{1}{2}\right)^{x-5}$
7. $f(x) = \left(\frac{1}{2}\right)^{6(x+4)}$
8. $f(x) = \left(\frac{1}{2}\right)^{3x+3}$
9. Graph the exponential function $f(x) = \left(-\frac{1}{5}\right)\left(10^{\frac{1}{2}(x+3)}\right)$ as a transformation of $f(x) = 10^x$.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## 12.4 Solving Exponential Equations Algebraically (DOK 2)

The properties of exponents can be used to solve exponential equations.

**Example 1:** Solve the exponential equation $4^{5(2+x)} = 1024$ for $x$.

**Step 1:** Rewrite the equation, so that the bases on both sides of the equation are the same.

$4^{5(2+x)} = 1024$

$4^{5(2+x)} = 4^5$

**Step 2:** If the bases on both sides of the equation are the same, then the exponents must be equal.

$5(2 + x) = 5$

**Step 3:** Solve for $x$.

$5(2 + x) = 5$

$2 + x = 1$

$x = -1$

**Example 2:** Solve the exponential equation $4^{x+1} = 32^{2x}$ for $x$.

**Step 1:** Rewrite the equation so that the bases on both sides of the equation are the same. Then simplify.

$4^{x+1} = 32^{2x}$

$\left(2^2\right)^{x+1} = \left(2^5\right)^{2x}$

$2^{2(x+1)} = 2^{5(2x)}$

$2^{2x+2} = 2^{10x}$

**Step 2:** If the bases on both sides of the equation are the same, then the exponents must be equal.

$2x + 2 = 10x$

**Step 3:** Solve for $x$.

$2x + 2 = 10x$

$8x = 2$

$x = \frac{1}{4}$

**Solve each of the following exponential equations. (DOK 2)**

1. $2^x = 64$

2. $6^{x+5} = 216$

3. $4^{1-x} = 512^{x-1}$

4. $9^{-4(x+2)} = 6{,}561^{2x+1}$

5. $(3)(8^x) = 96$

6. $9^{x+7} = 27^{3x}$

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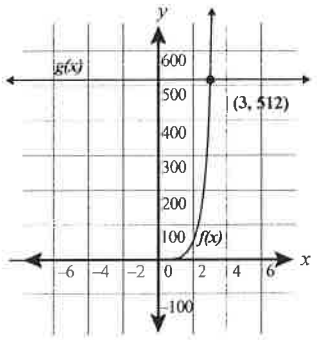
## 12.5 Solving Exponential Equations Graphically (DOK 2, 3)

To solve a simple exponential equation graphically, consider one side of the equation as one function and the other side of the equation as another function. Then graph the two functions and see where they intersect.

**Example 1:** Solve the equation $2^{3x} = 512$ graphically.

**Step 1:** To solve the equation, the functions $f(x) = 2^{3x}$ and $g(x) = 512$ should be graphed. Since the function $f(x) = 2^{3x}$ is in the form $f(x) = 2^{cx}$, with $c$ equal to 3, the graph of the function $f(x) = 2^x$ should be horizontally compressed by a factor of $\frac{1}{3}$.

The graphs of $f(x) = 2^{3x}$ and $g(x) = 512$ are as follows:




**Step 2:** Find where the two functions intersect.

Here the functions intersect at the point (3,512). So, when $x = 3$ the equation is true.

**Graph each of the following exponential equations to find the solution, then check your answer analytically. Use your own graph paper. (DOK 2)**

1. $5^{2x} = 3{,}125$
2. $6^{4(x-6)} = \frac{1}{1{,}296}$
3. $(-8)(12x) + 5 = -13{,}819$
4. $5^{\sqrt{5x}} = 3{,}125$

**Answer the question below. (DOK 3)**

5. Why does the graph of the function $f(x) = 2^{\frac{1}{x}}$ get closer and closer to the $y$-axis but never touch it?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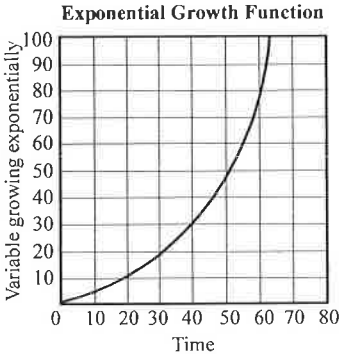
## 12.6 Real-World Exponential Functions (DOK 2)

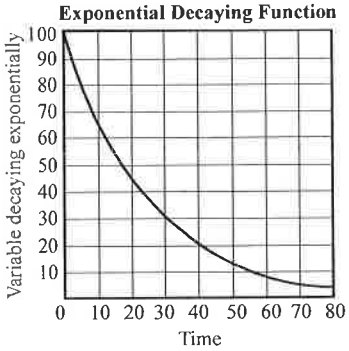
Real-life examples can be represented by functions. The most common functions are exponential growth and decay and half-life. Many quantities experience exponential growth or decay under certain conditions. Examples include bacteria, populations, disease, money in a savings account that compounds interest, and radioisotopes. Exponential functions are those functions in which the independent variable is time, and time is an exponent (thus the name exponential function). For instance, the formula for growth of money in a savings account that compounds interest annually is as follows:

$$A = P(1 + r)^t$$

where $A$ is the value of the account after $t$ years, $P$ is the original amount of money in the account, and $r$ is the annual interest rate.

Below are graphs of the general forms of exponential growth functions and exponential decay functions. Time is represented on the $x$-axis. Whatever is growing or decaying exponentially, such as population or money, is represented on the $y$-axis. Note that exponential function graphs are generally in Quadrant I since time objects cannot be assigned negative values.




**Example 1:** A city has a population of about 410,000 people. The US Census Bureau estimates that the population will double in 26 years. If the population continues at the same rate, what will the population be in
A) 10 years?
B) 50 years?

**Step 1:** Use the double growth equation $P = P_0(2^{t/d})$, where $P$ = population at time $t$, $P_0$ = population at time $t = 0$, and $d$ = double time.

**Step 2:** Determine the variable of each of the facts given in the problem. In this case, $P_0 = 410{,}000$ people, $d = 26$ years, and $t = 10$ years for part A, and $t = 50$ years for part B.

**Step 3:** Plug all of the information into the given equation. Round to the nearest whole number.
A) $P = 410{,}000(2^{10/26}) = 410{,}000\ (1.3055) = 535{,}260$ people
B) $P = 410{,}000(2^{50/26}) = 410{,}000\ (3.7923) = 1{,}554{,}847$ people

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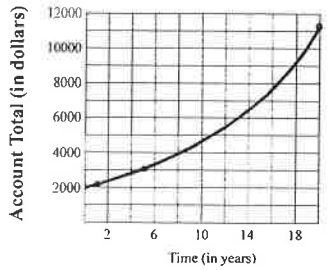
**Example 2:** Mason deposited $2,000 into a savings account that pays an annual interest rate of 9% compounded annually. Using the formula $A = P(1 + r)^t$ determine the amount of money in the savings account after 1 year, 5 years, and 20 years. Using the calculated values, construct a graph.

**Step 1:** Consider the known values. $P = 2{,}000$, and $r = 0.09$. The problem will have to be worked three times where $t = 1$, $t = 5$, and $t = 20$. $A$ is the amount being calculated.

**Step 2:**

| | | |
|---|---|---|
| $A = 2000(1+0.09)^1$ | $A = 2000(1+0.09)^5$ | $A = 2000(1+0.09)^{20}$ |
| $A = 2000(1.09)^1$ | $A = 2000(1.09)^5$ | $A = 2000(1.09)^{20}$ |
| $A = \$2{,}180$ | $A = \$3{,}077.25$ | $A = \$11{,}208.82$ |

**Step 3:** Use the calculated values to graph the function.




**Find the answers by using the equations and variables given. Round your answer to the nearest whole numbers. (DOK 2)**

**For question 1, use the following half-life formula.**

$$A = A_0\left(\frac{1}{2}\right)^{t/h}$$

$A$ = amount at time $t$ $\quad$ $A_0$ = amount at time $t = 0$ $\quad$ $h$ is the half-life

1. Chlorine (Cl) has a half-life of 55.5 minutes. If you start with 200 milligrams of chlorine, how many will be left after 5 hours?

**For questions 2, use the double growth formula. (DOK 2)**

$$P = P_0(2)^{t/d}$$

$P$ = amount at time $t$ $\quad$ $P_0$ = amount at time $t = 0$ $\quad$ $d$ is the half-life

2. There are about 3,390,000 Girl Scouts in the United States. The Girl Scout Council says that there is a growth rate of 5–10% per year, so they expect the Girl Scout population in the United States to double in 12 years. If the Girl Scout's organization expands as continuously as it has been, what will the population be

   A) in 8 years?

   B) next year?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**For question 3, use the compound interest formula. (DOK 2)**

$$A = P\left(1+\frac{r}{k}\right)^{kt}$$

$A$ = amount at time, $t$

$P$ = principle amount invested

$k$ = how many times per year interest is compounded

$r$ = rate

3. Lisa invested $1,000 into an account that pays 6% interest compounded monthly. If this account is for her newborn boy, how much will the account be worth on his 21st birthday, which is exactly 21 years from now? Assume the interest rate stays the same.

**Fill in the tables for the following functions. On the line under each table, label the given function as an exponential growth function or an exponential decay function. Round your answers to two decimal places. For extra practice, graph the functions. (DOK 2)**

4. $M(t) = 1000(1.04)^t$

| $t$ | $M(t)$ |
|---|---|
| 2 | |
| 4 | |
| 6 | |
| 8 | |

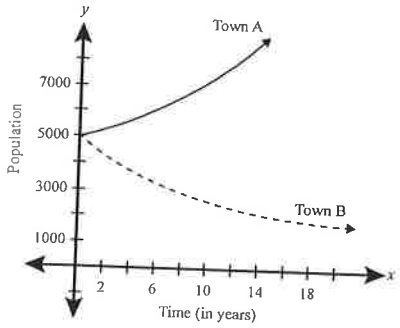
_____

5. $C(t) = 5300(0.5)^t$

| $t$ | $C(t)$ |
|---|---|
| 5 | |
| 10 | |
| 15 | |
| 20 | |

_____

**Refer to the graph to answer the following questions. (DOK 2, 3)**




6. Which town is experiencing exponential decay? Growth?

7. Why would it not make sense to draw the graph of town B below the $x$-axis?

8. In what year does the population of town B reach 3,000?

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## CHAPTER 12 REVIEW

**For each of the following exponential functions, list the function's domain, range, and zeros, and for the graph of the function, list the asymptotes, intercepts, intervals of increase/decrease, and end behavior. (DOK 3)**

1. $f(x) = 9^x$

2. $f(x) = \left(\frac{1}{7}\right)^{-x}$

3. $f(x) = (8)(4^x) + 6$

4. $f(x) = (-5)(3^x) - 1$

**Find the average rate of change of each of the following functions from $x = 1$ to $x = 3$. (DOK 2)**

5. $f(x) = (16)(5^x) + 3$

6. $f(x) = \left(\frac{7}{10}\right)\left(2^x\right) - 5$

7. $f(x) = (-4)(3^x) + 2$

**Use your own paper to graph each of the following exponential functions as transformations of $f(x) = 7^x$. (DOK 2)**

8. $f(x) = (-2)(7^x) - 3$

9. $f(x) = \left(\frac{1}{8}\right)\left(7^x\right) + 2$

10. $f(x) = 7^{5(x+1)}$

11. $f(x) = 7^{\frac{1}{2}(x-7)}$



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**Solve each of the following exponential equations. (DOK 2)**

12. $10^{x+1} = 100$

13. $5^{3(x-9)} = 125$

14. $(-8)(6^x) = -288$

15. $4^{x+8} = 322x$

16. $9^{x-50} = 19{,}683^{3x}$

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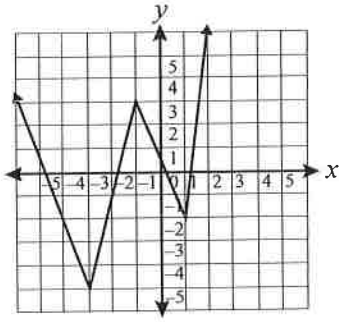
# Chapter 13
# Graphing Functions

Standards covered: F.IF.B.4, F.IF.B.5, F.IF.B.6, F.BF.B.3, F.LE.A.2

## 13.1 Evaluating Functions Using Graphs (DOK 1)

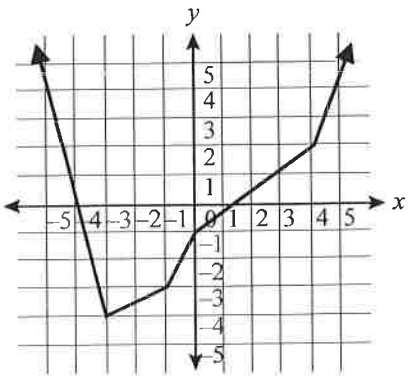
Functions can be evaluated by looking at its graph. Let's see how this is done in the following example.

**Example 1:** The graph of the function $y = f(x)$ is shown below. What is $f(2)$?




The question is asking what is the value of $y$, or $f(x)$, when $x$ is 2 on the graph given. First, find when $x$ is 2 and then find the $y$. On this graph, when $x$ is 2, $y$ is 5. Therefore, $f(2) = 5$.

**Use the graph of $y = f(x)$ below to answer questions 1–4. (DOK 1)**




1. What is $f(-3)$?
2. What is $f(4)$?
3. What is $f(-5)$?
4. What is $f(0)$?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## 13.2 Graphing Parent Functions (DOK 2)

A graph is an image that shows the relationship between two or more variables. In this section, we will learn how to graph six basic parent functions.

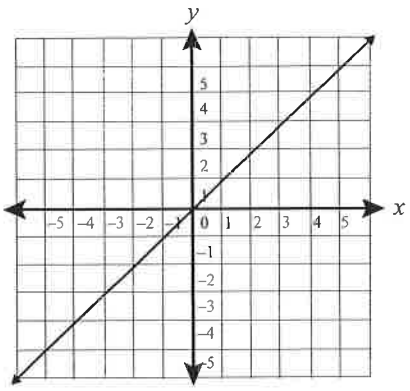
**Example 1:** $f(x) = x \rightarrow$ **linear parent function**

**Note:** The notation $f(x)$ is the same as $y$, it just means $f$ as a function of $x$.

The linear function, $y = x$, is called the **linear parent function**. If the graph of any function is a straight line, then its parent function is $y = x$.

The easiest way to begin graphing this is to draw a table of values. To create an accurate graph, you should choose at least 5 values for your table. These values are now your $(x, y)$ ordered pair and they are ready to be plotted.

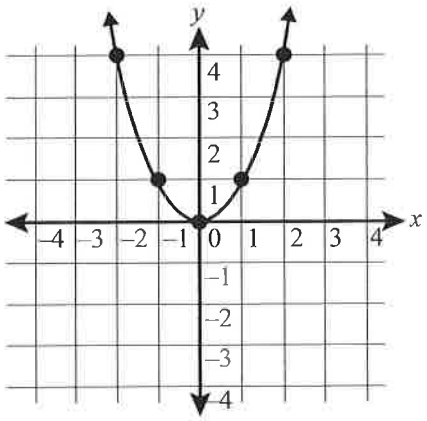
| $x$ | $f(x)$ |
|---|---|
| −2 | −2 |
| −1 | −1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |




**Example 2:** $f(x) = x^2 \rightarrow$ **quadratic parent function**

If an equation can be written in the form $y = ax^2 + bx + c$, where $a$, $b$, and $c$ are constants and $a \neq 0$, then it is a **quadratic equation**. The parent function is $y = x^2$. Also, if the graph of any function is a parabola as shown, then its parent function is $y = x^2$.

| $x$ | $f(x)$ |
|---|---|
| −2 | 4 |
| −1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |

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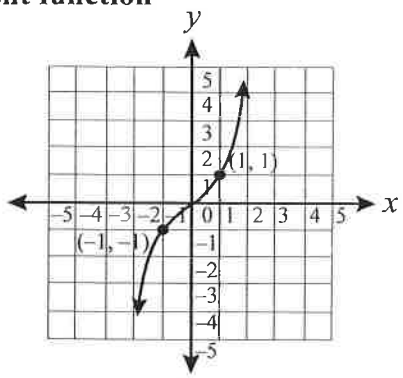
**Example 3:** $f(x) = x^3$ **– cubic parent function**

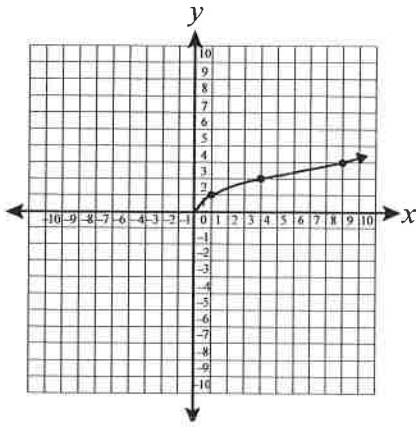
| $x$ | $f(x)$ |
|---|---|
| −2 | −8 |
| −1 | −1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 8 |




**Example 4:** $f(x) = \sqrt{x}$ → **square root parent function**

An equation like this will be easier to graph if you choose values of $x$ that are perfect squares, and because you can't take the square root of a negative, there is no need to select negative values.

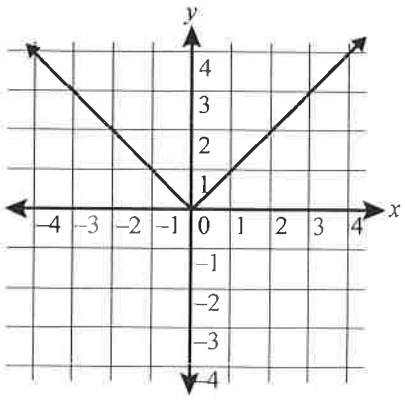
| $x$ | $f(x)$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 4 | 2 |
| 9 | 3 |




**Example 5:** $f(x) = |x|$ → **absolute value parent function**

**Note:** Don't forget, when you take the absolute value of a negative number, it becomes positive!

| $x$ | $f(x)$ |
|---|---|
| −2 | 2 |
| −1 | 1 |
| 0 | 0 |
| 1 | 1 |
| 2 | 2 |



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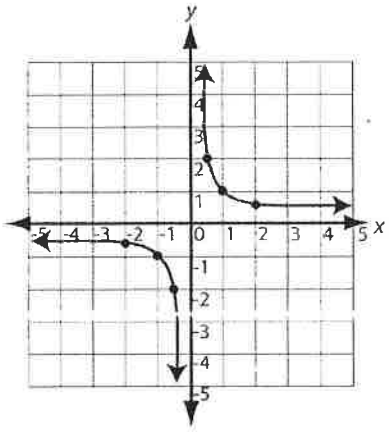
**Example 6:** $f(x) = \frac{1}{x}$ → **inverse parent function**

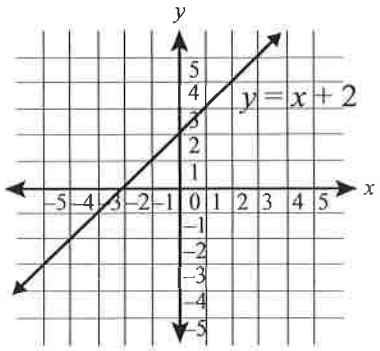
**Note:** Don't forget, anything over zero is undefined and therefore does not exist!

| $x$ | $f(x)$ |
|---|---|
| $-2$ | $-\frac{1}{2}$ |
| $-1$ | $-1$ |
| $-\frac{1}{2}$ | $-2$ |
| $0$ | Does not exist |
| $\frac{1}{2}$ | $2$ |
| $1$ | $1$ |
| $2$ | $\frac{1}{2}$ |



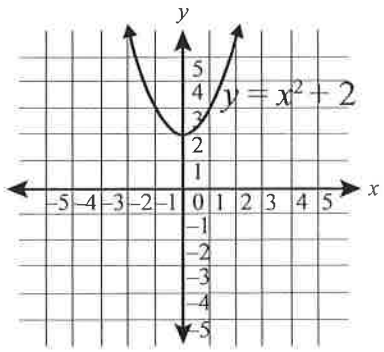
Now, let's look at some graphs to determine their parent functions.

**Example 7:** What is the parent function of the following graph?




**Answer:** Since the graph is a straight line, the parent function is $y = x$, the linear parent function.

**Example 8:** What is the parent function of this graph?




**Answer:** Since the graph is a parabola, its parent function is the quadratic parent function $y = x^2$. We also will notice that the equation is $y = x^2 + 2$. By the definition, this equation is quadratic, making its parent function $y = x^2$.

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**Graph the following on your own graph paper. (DOK 2)**

1. $f(x) = 2x$
2. $f(x) = 3x^3$
3. $f(x) = -\left|\frac{1}{2}x\right|$
4. $f(x) = \sqrt{2x}$
5. $f(x) = \frac{6}{x}$
6. $f(x) = \frac{1}{3}x^2$

**Name the parent function of the following equations. (DOK 1)**

7. $y = 10x$
8. $y = x^2 - 4$
9. $y = -\frac{1}{3}x^3 + 8$
10. $y = |4x| - 2$

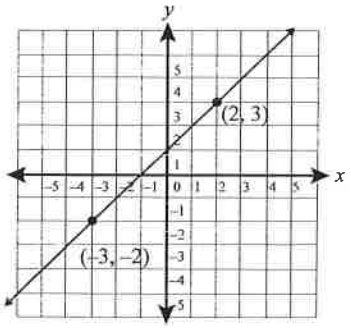
## 13.3 Comparing Rates of Change (DOK 1)

The rate of change is the value at which the quantity, $y$, is changing in respect to quantity, $x$.

$$\text{Rate of Change} = \text{change in } y \div \text{change in } x \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

In a linear function, the rate of change, also known as the slope, remains constant. This is great for describing an object moving at a constant speed, but that is not always the case. That is why, in quadratic functions, the rate of change varies. Quadratic equations have a variable rate of change.

**Example 1:** Linear Function, $y = x$




Find the slope: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-2)}{2 - (-3)} = \frac{5}{5} = 1$

This value shows that every time $x$ increases by one, $y$ does the same. The slope of a linear equation is always constant.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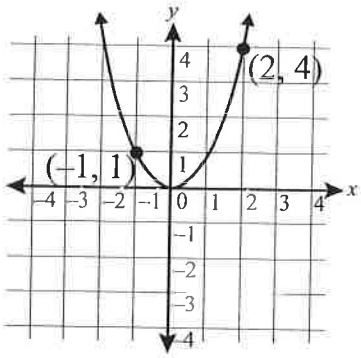
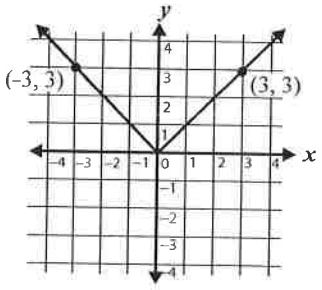
**Example 2:** Quadratic Function, $y = x^2$




Find the slope: $\frac{4-1}{2-(-1)} = \frac{3}{3} = 1$

This value can change depending on the points you choose, so the slope of a quadratic equation is variable.

**Example 3:** Absolute Value Function $y = |x|$




Find the slope: $\frac{3-3}{3-(-3)} = \frac{0}{6} = 0$

This, like the quadratic function, is constantly changing depending on the points you choose. It has a variable rate of change.

**Graph on your own paper and tell whether the slope is constant (c) or variable (v). (DOK 1)**

1. $f(x) = 3x + 2$
2. $f(x) = -x^2 + 4$
3. $f(x) = |x| + 7$
4. $f(x) = \frac{1}{2}x^2 - 1$
5. $f(x) = \sqrt{x}$

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## 13.4 Transformations of Graphs (DOK 2)

Graph transformation occurs when a graph is altered from its original, or parent, state. Graphs can be altered in 5 ways: shifting vertically, shifting horizontally, stretching, shrinking, or reflecting.

$$y = f(x + a) + b$$

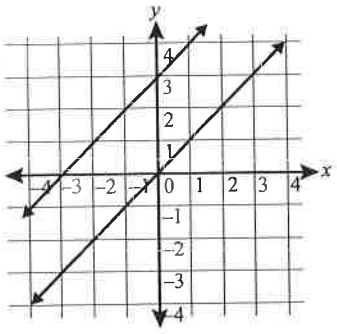
$a$ = horizontal shift (left/right), $b$ = vertical shift (up/down)

### Shifting Upward and Downward

**Example 1:** Given the graph of $f(x)$, what is the graph of function $y = f(x) + 3$?

$f(x) = x, y = x + 3$

The graph of function $y = f(x) + 3$ shifts the graph of $f(x)$ up 3 units. This transformation is called a **vertical shift**.

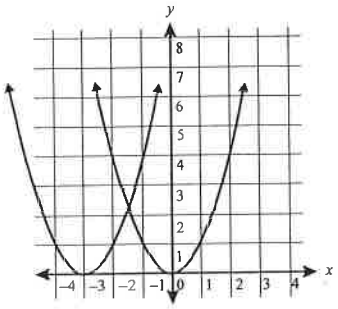



### Shifting Left and Right

**Example 2:** Given the graph of $f(x)$, what is the graph of function $y = f(x + 3)$?

$f(x) = x^2, y = (x + 3)^2$

The graph of function $y = f(x + 3)$ shifts the graph of $f(x)$ left 3 units. This transformation is called a **horizontal shift**.

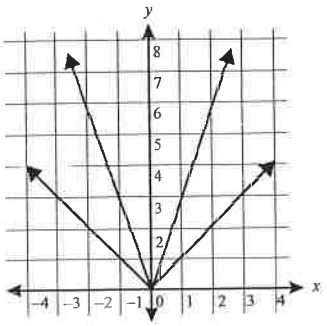



### Vertical Stretching and Shrinking

**Example 3:** Given the graph of $f(x)$, what is the graph of function $y = 3f(x)$?

$f(x) = |x|, y = 3|x|$

The graph of function $y = 3f(x)$ stretches the graph of $f(x)$ vertically. This transformation is called **stretching**.




**Note:** If $c > 1$, then the graph of $y = cf(x)$ is the graph of $f(x)$ stretched vertically by the ratio of $c$. If $0 < c < 1$, then the graph of $y = cf(x)$ is the graph of $f(x)$ compressed (shrunk) vertically by the ratio of $c$.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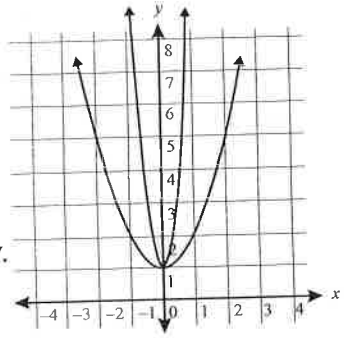
### Horizontal Stretching and Shrinking

**Example 4:** Given the graph of $f(x)$, what is the graph of function $y = f(3x)$?



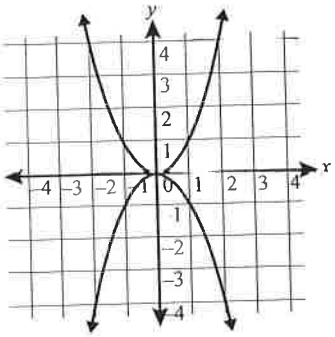
$f(x) = x^2 + 1, y = 9x^2 + 1$

The graph of function $y = f(3x)$ shrinks the graph of $f(x)$ horizontally. This transformation is called **shrinking**.

**Note:** If $c > 1$, then the graph of $y = f(cx)$ is the graph of $f(x)$ shrunk horizontally by the ratio of $c$. If $0 < c < 1$, then the graph of $y = f(cx)$ is the graph of $f(x)$ stretched horizontally by the ratio of $c$.

### Reflection across the $x$-axis

**Example 5:** Given the graph of $f(x)$, what is the graph of function $y = -f(x)$?

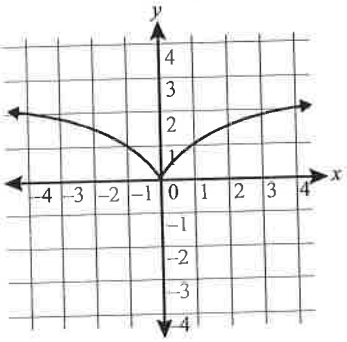


$f(x) = x^2, y = -x^2$

The graph of function $y = -f(x)$ reflects the graph of $f(x)$ across the $x$-axis. This transformation is called a **reflection**.

### Reflection across the $y$-axis

**Example 6:** Given the graph of $f(x)$, what is the graph of function $y = f(-x)$?



$f(x) = \sqrt{x}$, $y = \sqrt{-x}$

The graph of function $y = f(-x)$ reflects the graph of $f(x)$ across the $y$-axis. This transformation is called a **reflection**.

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Here are some tables for you to review how transformations of graphs occur.

| **Vertical Transformations (*outside* of parentheses, absolute values, square roots, etc.)** | | | |
|---|---|---|---|
| | **Examples** | | **Direction** |
| Positive (+) Translation | $f(x) = (x) + 1$<br>$f(x) = (x)^2 + 1$<br>$f(x) = (x)^3 + 1$ | $f(x) = \sqrt{x} + 1$<br>$f(x) = \lvert x \rvert + 1$<br>$f(x) = \frac{1}{x} + 1$ | Move **up** number of units. |
| Negative (–) Translation | $f(x) = (x) - 1$<br>$f(x) = (x)^2 - 1$<br>$f(x) = (x)^3 - 1$ | $f(x) = \sqrt{x} - 1$<br>$f(x) = \lvert x \rvert - 1$<br>$f(x) = \frac{1}{x} - 1$ | Move **down** number of units. |
| Stretch (Coefficient is whole number >1.) | $f(x) = 2(x)$<br>$f(x) = 2(x)^2$<br>$f(x) = 2(x)^3$ | $f(x) = 2\sqrt{x}$<br>$f(x) = 2\lvert x \rvert$<br>$f(x) = 2\left(\frac{1}{x}\right)$ | **Multiply** each $y$-coordinate by multiple of coefficient. |
| Shrink (Coefficient is <1.) | $f(x) = \frac{1}{2}(x)$<br>$f(x) = \frac{1}{2}(x)^2$<br>$f(x) = \frac{1}{2}(x)^3$ | $f(x) = \frac{1}{2}\sqrt{x}$<br>$f(x) = \frac{1}{2}\lvert x \rvert$<br>$f(x) = \frac{1}{2}\left(\frac{1}{x}\right)$ | **Multiply** each $y$-coordinate by multiple of coefficient. |



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| Horizontal Transformations (*inside* of parentheses, absolute values, square roots, etc.) | | | |
|---|---|---|---|
| | **Examples** | | **Direction** |
| Positive (+) Translation | $f(x) = (x + 3)$<br>$f(x) = (x + 3)^2$<br>$f(x) = (x + 3)^3$ | $f(x) = \sqrt{x+3}$<br>$f(x) = \lvert x + 3 \rvert$<br>$f(x) = \frac{1}{x+3}$ | Move **left** number of units. |
| Negative (–) Translation | $f(x) = (x - 4)$<br>$f(x) = (x - 4)^2$<br>$f(x) = (x - 4)^3$ | $f(x) = \sqrt{x} - 1$<br>$f(x) = \lvert x - 4 \rvert$<br>$f(x) = \frac{1}{x-4}$ | Move **right** number of units. |
| Stretch (Coefficient is whole number <1.) | $f(x) = \left(\frac{1}{5}x\right)$<br>$f(x) = \left(\frac{1}{5}x\right)^2$<br>$f(x) = \left(\frac{1}{5}x\right)^3$ | $f(x) = \sqrt{\frac{1}{5}x}$<br>$f(x) = \left\lvert\frac{1}{5}x\right\rvert$<br>$f(x) = \frac{1}{\frac{1}{5}x}$ | **Divide** each $x$ by number of coefficient. |
| Shrink (Coefficient is >1.) | $f(x) = (5x)$<br>$f(x) = (5x)^2$<br>$f(x) = (5x)^3$ | $f(x) = \sqrt{5x}$<br>$f(x) = \lvert 5x \rvert$<br>$f(x) = \frac{1}{5x}$ | **Divide** each $x$ by number of coefficient. |

**Complete the following. (DOK 2)**

1. How is $y = x - 7$ different from its parent graph?

2. How does $y = 3x^2$ differ from its parent graph?

3. Does $y = -f(x)$ reflect across the $x$ or $y$-axis?

4. Does $y = f(-x)$ reflect across the $x$ or $y$-axis?

5. How would adding 3 to $y = x^2$ change the graph?

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6. Would $y = f(3x)$ shrink the graph of $f(x)$ horizontally or vertically?

7. What is the parent function of $y = x + 12$?

8. Would $y = |x| - 7$ be shifted vertically or horizontally from its parent function?

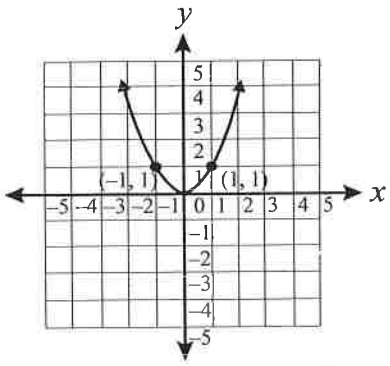
## 13.5 Symmetry of Graphs of Functions (DOK 2)

Symmetry with respect to the $y$-axis, $x$-axis, or origin can be determined by looking at the graph. If you are unable to tell just by looking at the graph, you can check your answers using the following:

| | |
|---|---|
| $y$-axis (even) | If Point $(a, b)$ is on the graph, so is Point $(-a, b)$. |
| origin (odd) | If Point $(a, b)$ is on the graph, so is Point $(-a, -b)$. |
| $x$-axis | If Point $(a, b)$ is on the graph, so is Point $(a, -b)$. |

**Example 1:** Determine if the graph of $y = x^2$ is symmetrical. If it is, is it odd, even, or neither?

**Step 1:** Graph the function.




Looking at the graph we see that the graph is symmetrical about the $y$-axis. In other words, whatever the graph shows on one side of the $y$-axis, it is the same on the other side of the $y$-axis.

**Step 2:** We check using the points on the graph. We will use the point (1, 1), as it is a solution to the equation.

According to the table above, if (1, 1) is a solution to a graph that is symmetrical about the $y$-axis, then $(-1, 1)$ must also be a solution.

Is it? $y = (-1)^2 = 1$. Yes, it is.

We also know since the graph is symmetrical about the $y$-axis that the graph is even.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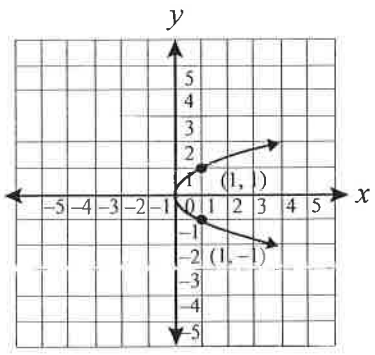
**Example 2:** Determine if the graph of $x = y^2$ is symmetrical? If it is, is it odd, even, or neither?

**Step 1:** Graph the function.

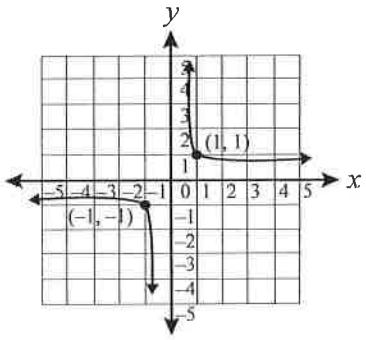



Looking at the graph we see that it is symmetrical about the $x$-axis. In other words, whatever the graph shows on one side of the $x$-axis, it is the same on the other side of the $x$-axis.

**Step 2:** We check using the points on the graph. We will use the point (1, 1), as it is a solution to the equation. If (1, 1) is a solution to a graph that is symmetrical about the $x$-axis, then (1, −1) must also be a solution. Is it? Yes.
We also know since the graph is symmetrical about the $x$-axis that the graph is neither even or odd.

**Example 3:** Determine if the graph of $y = \frac{1}{x}$ is symmetrical? If it is, is it odd, even, or neither?

**Step 1:** Graph the function.




Looking at the graph we see that the graph is symmetrical about the origin. In other words, if you rotate one part of the graph 180° around the origin, it will be the exact same as the other half of the graph.

**Step 2:** We check using the points on the graph. We will use the point (1, 1), as it is a solution to the equation. If (1, 1) is a solution to a graph that is symmetrical about the origin, then (−1, −1) must also be a solution. Is it? Yes.

We also know since the graph is symmetrical about the origin that the graph is odd.

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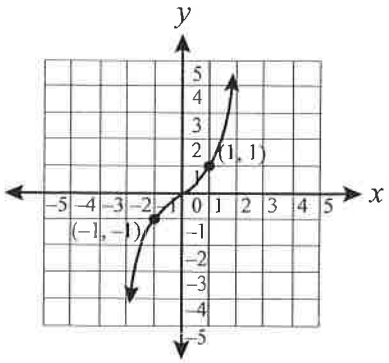
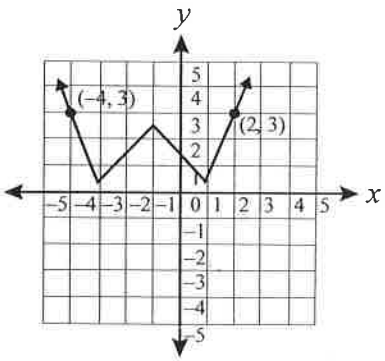
**Tell whether the graph is symmetrical with respect to the $x$-axis, $y$-axis, the origin, or neither. Also, tell whether the graph is odd, even, or neither. (DOK 2)**

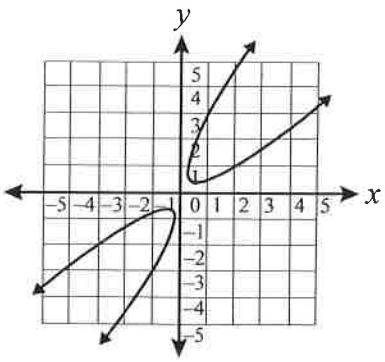
1.



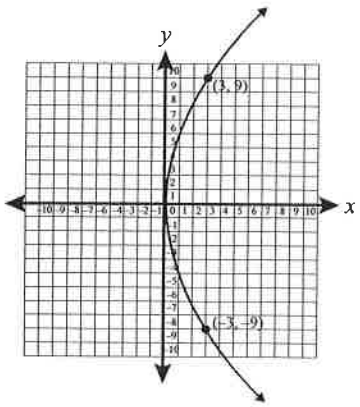

3.




2.




4.




## 13.6 Symmetry of Functions Algebraically (DOK 2)

To test equations for symmetry algebraically, it is helpful to remember the following:

$f(-x) = f(x)$ means the function is symmetrical with respect to the $y$-axis.

Being symmetrical with the $y$-axis means the function is **even**.

$f(-x) = -f(x)$ means the function is symmetrical with respect to the origin.

Being symmetrical with the origin means the function is **odd**.

$f(-x) \neq -f(x)$ or $f(x)$ then the function is not symmetrical.

**Example 1:** Test the following function for symmetry. $f(x) = x^4 + x^2 + 3$

**Step 1:** First, we need to substitute $-x$ in the function for $x$.

$f(-x) = (-x)^4 + (-x)^2 + 3$

**Step 2:** Carry out the operations.

$f(-x) = x^4 + x^2 + 3$



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**Step 3:** Since $f(-x) = x^4 + x^2 + 3$ and $f(x) = x^4 + x^2 + 3$, then $f(-x) = f(x)$.
This means $f(x) = x^4 + x^2 + 3$ is symmetrical with respect to the $y$-axis.
The function $f(x) = x^4 + x^2 + 3$ is even.

**Example 2:** Test the following function for symmetry. $f(x) = x^5 + x^3 + x$

**Step 1:** First, we need to substitute $-x$ in the function for $x$.
$f(-x) = (-x)^5 + (-x)^3 + (-x)$

**Step 2:** Carry out the operations.
$f(-x) = -x^5 - x^3 - x$

**Step 3:** Factor out a negative. $f(-x) = -x^5 - x^3 - x = -(x^5 + x^3 + x)$. Since $f(-x) = -(x^5 + x^3 + x)$ and $-f(x) = -(x^5 + x^3 + x)$, then $f(-x) = -f(x)$.
This means $f(x) = x^5 + x^3 + x$ is symmetrical with respect to the origin.
The function $f(x) = x^5 + x^3 + x$ is odd.

**Example 3:** Test the following function for symmetry. $f(x) = x^7 + x^4 - x^2$

**Step 1:** First, we need to substitute $-x$ in the function for $x$.
$f(-x) = (-x)^7 + (-x)^4 - (-x)^2$

**Step 2:** Carry out the operations.
$f(-x) = -x^7 + x^4 - x^2$

**Step 3:** In this case, we see that $f(-x) \neq -f(x)$ or $f(x)$, which means this function is neither even nor odd (no symmetry).

**Determine whether the function is even, odd, or neither. (DOK 2)**

1. $f(x) = x^3 + x^2 + x + 1$

2. $f(x) = 2x^3 - 4x$

3. $f(x) = 8x^2 + x^4$

4. $f(x) = 4x^3 + x$

5. $f(x) = 6x^3 + 2x^2 - x$

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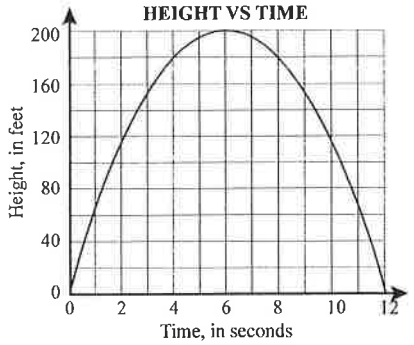
## 13.7 Real-World Functions (DOK 2)

Quadratic functions and their graphs are used to model many real-world situations. Some common examples are rockets that are launched, arrows shot into the air, golf balls hit into the air, and divers diving off of platforms. Instead of writing $y$ changing as a function of $x$, these variables are usually expressed as height changing as a function of time. The usefulness of these graphs is that, after a certain number of experiments, you can use the mathematical models to accurately predict future outcomes without performing any more experiments.

**Example 1:** An arrow was shot into the air. The graph shows the height, $h$, of its flight, $t$, seconds after it was shot.




**What can we say about this graph strictly from our knowledge of quadratic functions?** Time, $t$, in seconds is graphed along the $x$-axis. We know that the height of the arrow will depend on the time, so $t$ is the independent variable. The group of $t$ values will be the domain of the function as graphed along the $x$-axis, which describes the time that the arrow is in the air. Height, $h$, in feet, is graphed along the $y$-axis. Height, $h$, will be the dependent variable, and the group of all $h$ values will be the range of the function which describes the height of the flight path.

**What is the $x$-intercept of this graph?** Actually there are two places where this graph crosses the $x$-axis (or in this case, where $h = 0$). The first is at (0, 0). This is the point at which the arrow is on the ground prior to being shot. The second is at (12, 0). The arrow is back on the ground ($h = 0$) after flying for 12 seconds.

**What is the $y$-intercept?** This is the point where $x$ (or $t$ in this case) is equal to 0. At $t = 0$, $h = 0$. This means the $y$-intercept is prior to the arrow being shot at (0, 0).

**Does the function have a maximum or a minimum?** Yes. At $t = 6$ seconds, the arrow reaches a maximum height of 200 feet and begins descending. The minimum is when the arrow is at zero height and $t = 0$ and $t = 12$.

**Where does the arrow reach 100 feet?** Look closely. It reaches 100 feet the first time at $t = 2$ on the way up and again at $t = 10$ on the way down.

**What is the domain of the function?** The domain can be described as all the values of $t$ that make up the first coordinates of the ordered pairs of the graph. For this graph, $0 \leq t \leq 12$.

**What is the range of the function?** It is all the values of $h$ that make up the second coordinates of the ordered pairs of the graphs. In this example, the range is $0 \leq h \leq 200$.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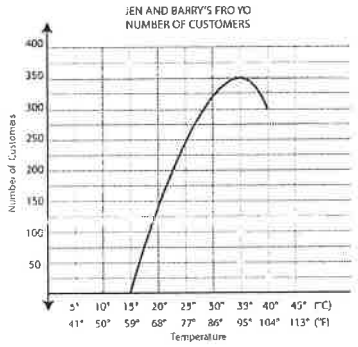
Real-world graphs of function have various forms.

**Example 2:** Jen and Barry run a frozen yogurt stand in the spring and summer months. The graph below shows the number of customers they have on a weekday as a function of the high temperature. They do not open when the temperature is less than 15 °C (59 °F) (too cold) or over 40 °C (104 °F) (too hot to work in the stand).



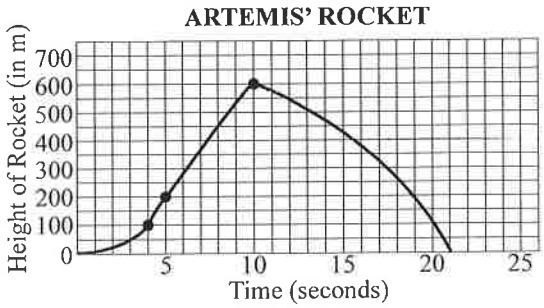

What is the maximum number of customers that Jen and Barry have for any given temperature? At what temperature do they have this many customers? What is the range of this function? The domain?

The maximum point on the curve is (35, 350), so the most customers that they will have is 350 customers. This maximum occurs at 35 °C (95 °F).

The $y$ values span from 0 customers to 350 customers, so the range is $0 \leq y \leq 350$, and the $x$ values span from 15 °C to 40 °C, so the domain is $15 \leq x \leq 40$.

**Use the graphs to answer the questions. (DOK 2, 3)**

Artemis and his friends shoot off a miniature rocket. It accelerates for 5 seconds, ascends at a steady pace for 5 seconds, then runs out of fuel and falls to the ground. The graph below shows the flight of the rocket.




1. According to the graph, the rocket was 100 m high after 4 seconds. Approximately when did it reach 100 m again?

   **A** 10 seconds **B** 20 seconds **C** 21 seconds **D** 25 seconds

2. What was the rate of increase (slope) between 5 seconds and 10 seconds?

   **A** 40 m/sec **B** 75 m/sec **C** 80 m/sec **D** 100 m/sec

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3. How long did the rocket's flight last? Write your answer as an ordered pair. This ordered pair represents what common characteristic of a graph? How do you know?

4. What is the maximum of this graph? What does the maximum represent in terms of the problem?

## Chapter 13 Review

**Use your own paper to graph each equation, and then state whether the slope is constant (c) or varying (v). (DOK 1)**

1. $f(x) = x - 6$

2. $f(x) = \frac{1}{2}x^2 + 6$

**Use what you know about graph transformations to answer the following questions. (DOK 1, 2)**

3. True or False: $y = f(-x)$ is reflected across the $y$-axis.

4. True or False: $y = 3f(x)$ shrinks a graph horizontally.

5. Write the equation of $y = x^2$ shifted down two units.

6. True or False: $y = f(2x)$ shrinks a graph horizontally.

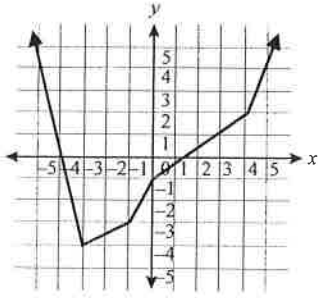
**Use your own paper to graph each function. (DOK 2)**

7. $f(x) = -|2x|$

8. $f(x) = \frac{1}{2}x^2$

9. $f(x) = \frac{1}{7}x$

**The graph of $y = f(x)$ is shown below. (DOK 2)**




10. What is $f(-1)$?

11. What is $f(5)$?



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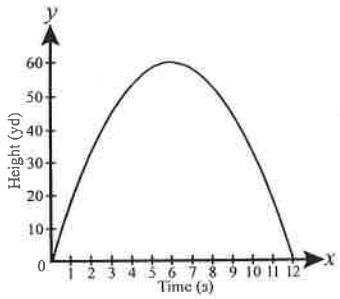
12. A rocket was shot up into the air. The graph shows the height $h$ of its flight $t$ seconds after it was shot.




At about what height was the rocket after 3 seconds? How long did it take the rocket to come back down to the ground?

**Solve the following problems about function symmetry. (DOK 1, 2)**

13. True or False. If for a function $f(x)$, $f(-x) = -f(x)$ is true, then the function is odd.

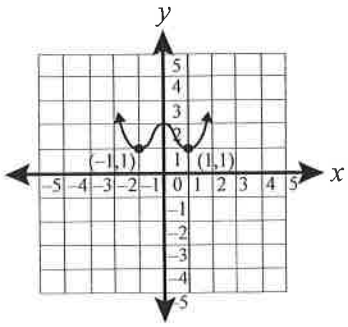
14. Fill in the blank. When all exponents in a function are _____, the function will be even.

15. Is the function $f(x) = x^6 + x^4 + 7$ even, odd, or neither?

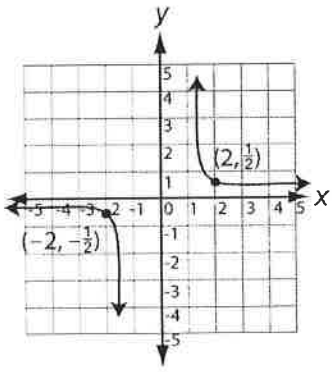
16. Is the function $f(x) = x^5 + x^3 + 2$ even, odd, or neither?

**State whether the graphs are symmetrical with respect to the $x$-axis, $y$-axis, origin, or neither. (DOK 2)**

17.




18.

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# Chapter 14
# Comparing Functions

Standards covered: F.IF.C.9, F.BF.A.1b, F.LE.A.1, F.LE.A.2, F.LE.A.3, F.LE.B.5

## 14.1 Constructing Functions (DOK 3)

**Example 1:** Find the equation of a line that passes through the points $(-3, -1)$ and $(1, 3)$. In order to find a line that passes through two points, we must use two points to find parts of the equation $y = mx + b$.

**Step 1:** Find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{3-(-1)}{1-(-3)}$$
$$= \frac{4}{4} = 1$$

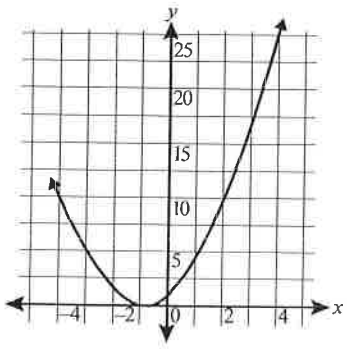
**Step 2:** Find the $y$-intercept by plugging in $x$ and $y$ values.

$y = x + b$
$3 = 1 + b$
$2 = b$

**Answer:** The function we construct is $y = x + 2$.

**Use what is given to find the equation of the function. (DOK 3)**

1.



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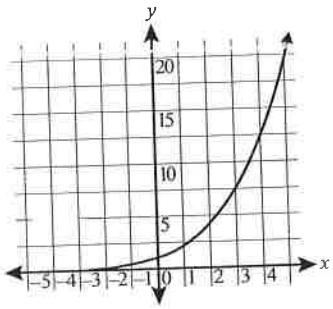
2. Exponential function that contains the points (0, 4) and (4, 19)

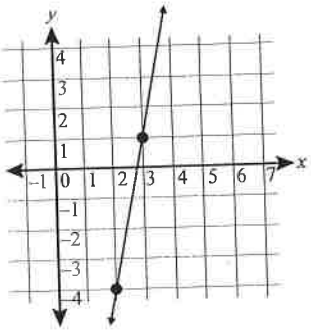
3. Linear function that contains the points (3, 19) and (7, 35)

4. Quadratic function with the vertex (−4, 3) and passes through the point (2, 39)

5.



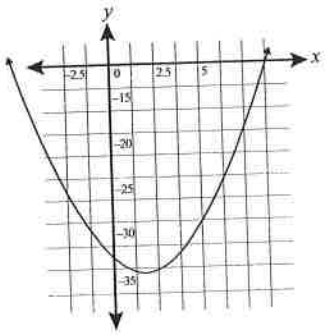

6.




## 14.2 Comparing Functions (DOK 2)

By comparing functions, you can tell a lot about each function.

**Example 1:** Given $f(x) = x^2 - 3x - 10$ and the graph below of $g(x)$, which function has a smaller minimum?

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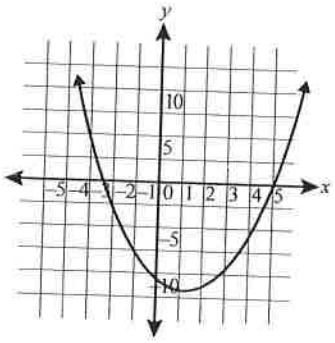
**Answer:** If we graph the $f(x)$ (below), we see that the graph of $g(x)$ (above) has a smaller minimum.



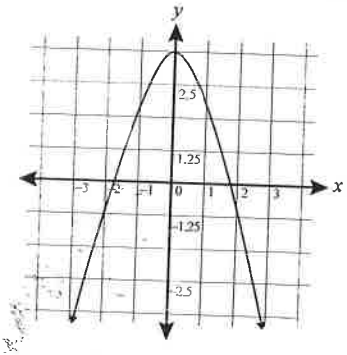

**In the following problems, you are given 2 different functions. Use the information provided to answer each problem. (DOK2)**

1. Given the function $f(x) = x^2 + 8x + 12$ and the table below, which function has the smaller minimum?

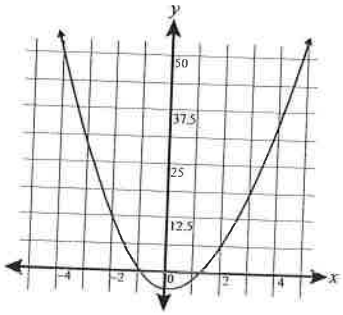
| $x$ | $g(x)$ |
|---|---|
| −3 | −18 |
| −2 | −11 |
| −1 | −2 |
| 0 | 9 |
| 1 | 22 |

2. Given the graph and the table below, which function has the greater rate of change?

| $x$ | $f(x)$ |
|---|---|
| −2 | −28 |
| −1 | −16 |
| 0 | −6 |
| 1 | 2 |
| 2 | 8 |




3. Given the function $f(x) = 2x^2 - 4x - 11$ and the graph of $g(x)$ (below), which function has the smaller minimum?



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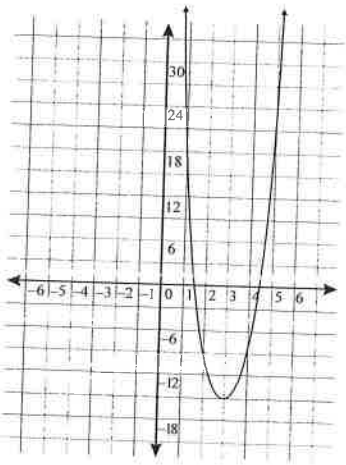
4. Given the function $f(x)=\begin{cases}2x+3 & \text{if } x<0\\ 2x-3 & \text{if } x\geq 0\end{cases}$ and the function $g(x)$ graphed below, which function has the most solutions at $y=0$?




## 14.3 Comparing Function Models (DOK 2)

What is the difference between the growth of a linear function and an exponential function? A linear function increases or decreases by equal differences per interval. This common difference is determined by the slope of the line. An exponential function increases or decreases by equal factors per interval; this common factor is determined by the coefficient of the exponential function.

**Example 1:** $y=x+3$ increases by 3 for every $y$

$y=3^x$ triples for every $y$

Now compare the two functions by looking at the tables below.

$y=x+3$

| $x$ | $y$ |
|---|---|
| 1 | 4 |
| 2 | 5 |
| 3 | 6 |
| 4 | 7 |
| 5 | 8 |

$y=3^x$

| $x$ | $y$ |
|---|---|
| 1 | 3 |
| 2 | 9 |
| 3 | 27 |
| 4 | 81 |
| 5 | 243 |

You can see that the exponential function grows at a much larger rate than the linear function and eventually exceeds the linear function.

When comparing functions, you can also distinguish which model will best represent a particular situation.

**Example 2:** Which model, linear or exponential, represents that if for every mile, a runner burns 100 calories?

**Answer:** The common difference per mile is 100 calories. Therefore, the function is linear.

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**Determine whether the following can be modeled by an exponential or a linear function. (DOK 2)**

1. Cancer cells triple every 6 months.

2. A paper plant must cut down 24 trees per ton of paper they make.

3. The number of visits to a website quadruples every month.

4. An intern makes $8.00 an hour.

## 14.4 Combining Functions (DOK 2)

In order to combine functions correctly, the operation must be observed very carefully. Some will ask you to add two functions together or could ask you to subtract from each other. Others may ask you to multiply or divide two functions together. There is also a combination strategy called composition which means inserting a function where the variable is in the other function and creating a whole new function.

**Example 1:** If $f(x) = 5x + 3$ and $g(x) = 2x^2 - 7$, then what is $f(x) + g(x)$?

**Step 1:** Substitute $f(x)$ and $g(x)$ with their appropriate functions into the expression.
$(5x + 3) + (2x^2 - 7)$

**Step 2:** Simplify the function expression by combining like terms. Remember that $x^2$ cannot be combined with $x$ since they are not the same exponent power, so this means that $-7$ and 3 are the only terms that can combine.
$2x^2 + 5x - 4$

**Answer:** $f(x) + g(x) = 2x^2 + 5x - 4$

**Example 2:** If $f(x) = 7x - 6$ and $g(x) = 3x + 4$, then what is $f(x) \cdot g(x)$?

**Step 1:** Substitute $f(x)$ and $g(x)$ with their appropriate functions into the expression.
$(7x - 6)(3x + 4)$

**Step 2:** Use the FOIL method to multiply the two expressions.
$21x^2 + 28x - 18x - 24$
Simplify the function expression by combining like terms.
$21x^2 + 10x - 24$

**Answer:** $f(x) \cdot g(x) = 21x^2 + 10x - 24$


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**Combine the following equations, paying careful attention to the order in which the functions are listed. (DOK 2)**

1. If $f(x) = 3x + 5$ and $g(x) = 2x + 4$, then what is $g(x) + f(x)$?

2. If $f(x) = x - 8$ and $g(x) = x + 8$, then what is $f(x) \times g(x)$?

3. If $f(x) = x^2 + 3x + 2$ and $g(x) = x + 1$, then what is $f(x) \div g(x)$?

4. If $f(x) = -3x + 4$ and $g(x) = x^2 - 7$, then what is $f(g(x))$?

5. If $f(x) = 5x + 7$ and $g(x) = x^2 - 7x + 4$, then what is $g(x) - f(x)$?

## 14.5 Evaluating Composed Functions at Given Values (DOK 2)

Some questions will ask to evaluate composed functions at certain values. Certain steps must be followed in order to correctly evaluate the functions.

**Example 1:** If $f(x) = 3x^2 + 7$ and $g(x) = 9x - 5$, then what is $f(g(-3))$?

**Step 1:** Just like composing equations, pay attention to the order listed. Evaluate the $g(x)$ function at $-3$.

$g(-3) = 9(-3) - 5$
$g(-3) = -27 - 5$
$g(-3) = -32$

**Step 2:** Substitute the value of $g(-3)$ into the $f(x)$ function where there is an $x$ variable.

$f(-32) = 3(-32)^2 + 7$
$f(-32) = 3(1024) + 7$
$f(-32) = 3072 + 7$
$f(-32) = 3079$

**Answer:** $f(g(-3)) = 3079$

**Evaluate the following functions at the given values. (DOK 2)**

1. If $f(x) = -2x^2 + 8$ and $g(x) = 7x^2 + 6$, then what is $f(g(2))$?

2. If $f(x) = 2x + 5$ and $g(x) = x^2 + 9$, then what is $g(f(1))$?

3. If $f(x) = 9x^2 + 7$ and $g(x) = -3x^2 - 8$, then what is $g(f(-4))$?

4. If $f(x) = 4x + 7$ and $g(x) = 7x^2$, then what is $f(g(3))$?

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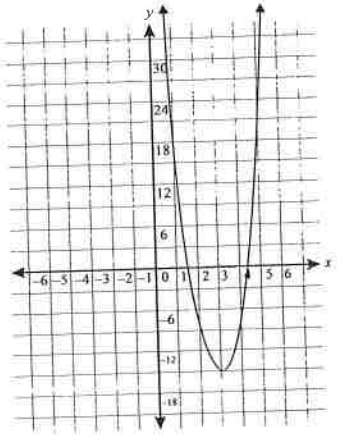
## Chapter 14 Review

**Answer the questions for each function below. (DOK 2)**

1. Given the function $f(x) = x^2 + 4x - 21$ and the table below, which function has the smaller minimum?

| $x$ | $g(x)$ |
|---|---|
| −2 | 13 |
| −1 | 0 |
| 0 | −11 |
| 1 | −20 |
| 2 | −27 |

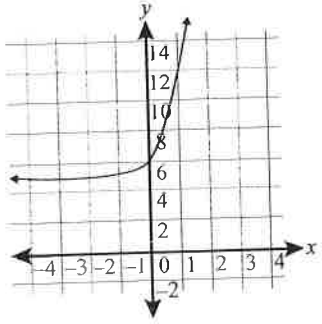
2. Given the function $f(x) = \begin{cases} 1 & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$ and the function $g(x)$ graphed below, which function has the greater $y$-intercept?




**Answer the questions about the functions below. (DOK 3)**

3. Find the quadratic function that has the vertex (–4, 5) and passes through the point (8, 21).

4. Find the exponential function that represents the function in the graph:





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**Determine whether the following can be modeled by an exponential or a linear function. (DOK 2)**

5. Lobster cost $18 per pound.

6. A bathtub drains at a rate of one gallon per minute.

7. A baby horse weighing 200 pounds will grow at a rate of 11% per month for the first year.

8. A $1500 deposit into a savings account that pays 7% interest every month.

**Combine the following equations, paying careful attention to the order in which the functions are listed. (DOK 2)**

9. If $f(x) = x^2$ and $g(x) = x + 4$, then what is $f(g(x))$?

10. If $f(x) = x - 3$ and $g(x) = x^2 + 4$, then what is $g(f(x))$?

**Evaluate the following functions at the given values. (DOK 2)**

11. If $f(x) = 7x - 3$ and $g(x) = 8x - 41$, then what is $g(f(2))$?

12. If $f(x) = 9x + 11$ and $g(x) = 3x + 11$, then what is $f(g(-2))$?

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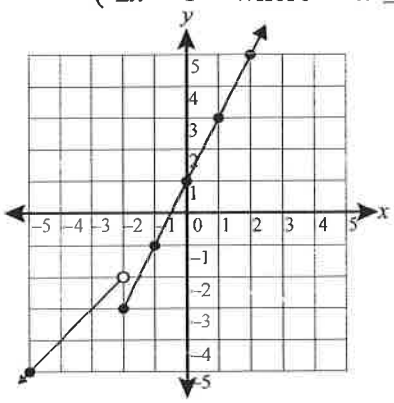
# Chapter 15
# Step and Piecewise Functions

Standards covered: F.IF.B.4

## 15.1 Step and Piecewise Functions (DOK 2)

There are a number of types of graphs that you may see which are not linear or quadratic.
A **piecewise** function is a function consisting of 2 or more formulas over a sequence of intervals.
These intervals are defined by the possible values of $x$, also known as the domain of the function.
The graph of a piecewise function consists of the graphs of each interval formula.

**Example 1:** $f(x) = \begin{cases} x & \text{where} \quad x < -2 \\ 2x + 1 & \text{where} \quad x \geq -2 \end{cases}$




Notice that there is a break in the graph at $x = -2$. Whenever you would have to pick up your pencil in order to draw the graph from left to right, the function is **discontinuous**. At every other point in a function, the function is **continuous**.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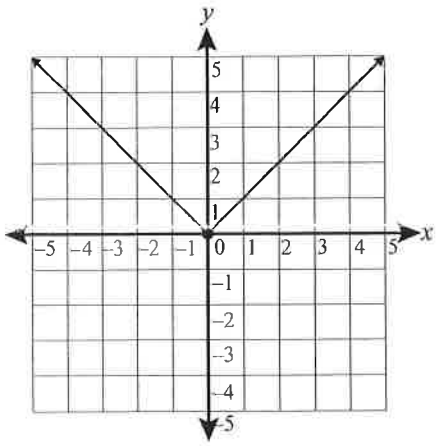
A particular type of piecewise defined function is the **absolute value** function. The absolute value of a number is its distance from 0. The absolute value of $x$ is written $|x|$.

**Example 2:** $f(x) = |x| = \begin{cases} -x & \text{where} \quad x < 0 \\ x & \text{where} \quad x \geq 0 \end{cases}$

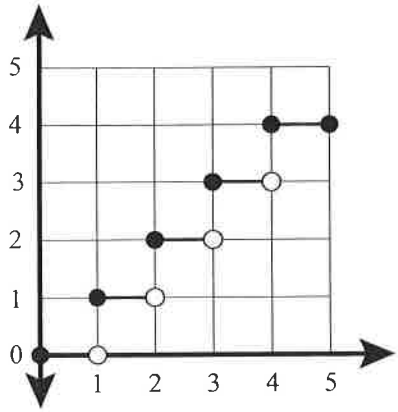



The absolute value function always gives a positive number, but it is **not** discontinuous at $x = 0$, because you do not have to pick up your pencil to draw it.

Another common type of piecewise defined function is a **step** function. Instead of a continuous increase or decrease of $y$ from left to right, the $y$ value stays the same for certain interval of $x$, then jumps to a higher or lower level. It therefore looks like a set of steps going up or down. When the inequality is less than or equal to ($\leq$), you must draw the endpoint as a filled in circle. This shows that the function includes that point. For the strict inequalities ($<$), you must draw an endpoint with an open (not filled in) circle.

**Example 3:** $f(x) = \begin{cases} 0 & \text{where} \quad 0 \leq x < 1 \\ 1 & \text{where} \quad 1 \leq x < 2 \\ 2 & \text{where} \quad 2 \leq x < 3 \\ 3 & \text{where} \quad 3 \leq x < 4 \\ 4 & \text{where} \quad 4 \leq x \leq 5 \end{cases}$

**Answer:**




The graph is discontinuous at every "step," in this case $x = 1$, $x = 2$, $x = 3$, and $x = 4$.

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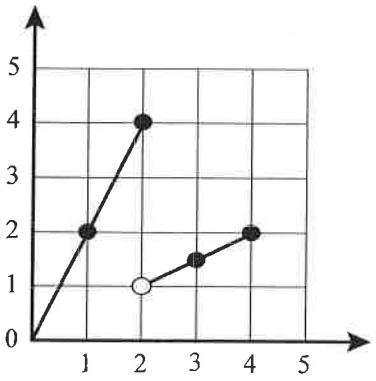
## 15.2 Characteristics of Piecewise Functions (DOK 2)

In addition to continuity, there are a number of features that you can identify from the graph of a nonlinear function.

**Zeros** are the points where $y = 0$. The zeros are the function's $x$-intercepts.
The **maximum** (plural: maxima) is the highest $y$ value that a function reaches.
The **minimum** (plural: minima) is the lowest $y$ value that a function reaches.
The **range** is all of the $y$ values that are defined by the function.
Similarly, the **domain** is all of the $x$ values defined by the function.
Finally, you can determine the **rate of increase or decrease** between two points by finding the rise over the run (slope).

**Example 4:** What are the zeros, maximum, minimum, range, and domain of the function below? At what point is the function discontinuous? Also, what is the rate of increase between $x = 0$ and $x = 2$.

$$y = \begin{cases} 2x & \text{where} \quad 0 \le x \le 2 \\ \frac{x}{2} & \text{where} \quad 2 < x \le 4 \end{cases}$$




If the graph contains a point, it is shown with a closed circle.

If the graph does not contain a point, like point (2, 1), you must use a open circle.

**Zeros:** The point where $y = 0$, or where graph intersects the $x$-intercept, is $x = 0$.

**Maximum:** Highest point is (2, 4). Since the maximum point of the graph is always the largest $y$ value that the graph reaches, 4 is the maximum.

**Minimum:** Lowest point is (0, 0). Since the minimum point of the graph is always the smallest $y$ value that the graph reaches, 0 is the minimum.

**Range:** Maximum − Minimum = 4 − 0 = 4

**Domain:** Highest $x$ − Lowest $x$ = 4 − 0 = 4

The function is discontinuous at $x = 2$ because this is the point where the graph is not connected.

**Rate of increase** between $x = 0$ and $x = 2$: At $x = 0$, $y = 0$. At $x = 2$, $y = 4$.

Therefore, the rise is 4 − 0 = 4, and the run is 2 − 0 = 2.

Slope = rise over run = $\frac{4}{2} = 2$.



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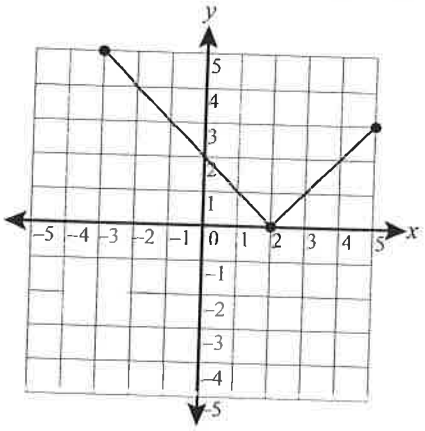
**Use the following graph of the absolute value function $y = |x - 2|$ for questions 1 and 2. (DOK 2)**




1. According to the graph, what is the minimum value of the function?

   **A** −3

   **B** 0

   **C** 2

   **D** 5

2. At what point is this graph discontinuous?

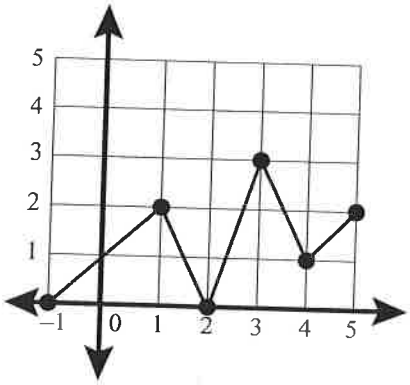
   **A** (−3, 5)

   **B** (0, 2)

   **C** (2, 0)

   **D** None of the above. The graph is continuous.

**Use the following graph of a piecewise function for questions 3 and 4. (DOK 2)**




3. How many zeros does this function have?

   **A** −1

   **B** 0

   **C** 1

   **D** 2

4. What are the range and domain of the function?

   **A** range: $0 \le y \le 3$, domain: $-3 \le x \le 5$

   **B** range: $-3 \le x \le 3$, domain: $0 \le y \le 5$

   **C** range: $-1 \le x \le 5$, domain: $0 \le y \le 3$

   **D** range: $0 \le y \le 3$, domain: $-1 \le x \le 5$

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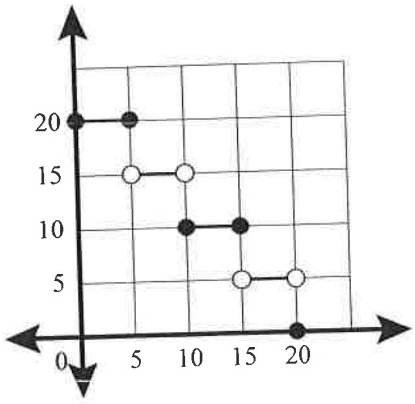
**Use the following graph of a step function for questions 5 and 6. (DOK 2)**




5. At how many points is this graph discontinuous?

**A** 4

**B** 8

**C** 6

**D** 2

6. What is the rate of increase of the function between 10 and 15?

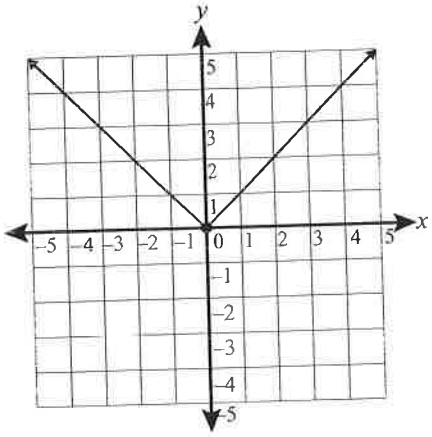
**A** −5

**B** 0

**C** 1

**D** 5

**Use the following graph of a step function for questions 7 and 8. (DOK 2)**




7. What is the vertex of this graph?

**A** (0, 0)

**B** (−5, 5)

**C** (5, 5)

**D** (0, 5)

8. What is the domain of this function?

**A** $-5 \leq x < 5$

**B** $0 \leq y < \infty$

**C** $0 \leq x < \infty$

**D** all real numbers



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## 15.3 More Piecewise Functions

A **piecewise function** is a function consisting of 2 or more formulas over a sequence of intervals. These **intervals** are defined by the possible values of $x$, also known as the domain of the function. The graph of a piecewise function consists of the graphs of each interval formula.

**Example 5:** $f(x) = \begin{cases} 3 & \text{if } 0 \le x < 1 \\ 2 & \text{if } 1 \le x < 2 \\ 1 & \text{if } 2 \le x < 3 \end{cases}$

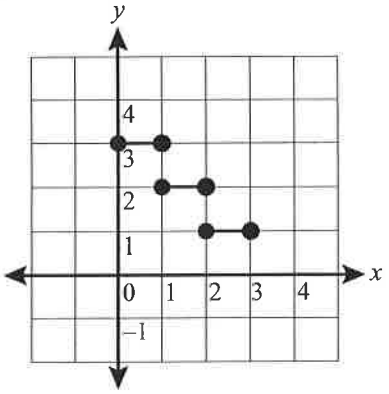
Graph $f(x)$.

**Step 1:** Graph each formula over the given interval.

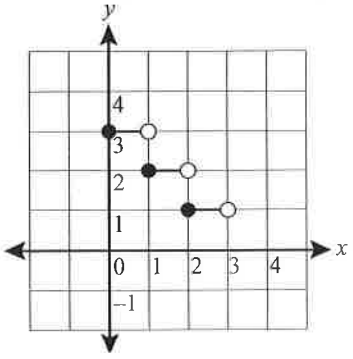
For example, $f(x) = 3$ when the domain is $0 \le x < 1$. This means that you would draw the graph $y = 3$ first. Recall that this is a horizontal line segment that passes through the point (0, 3). After this, you would only draw $y = 3$ between the points (0, 3) and (1, 3) because of the domain. The graph cannot go outside of those points.

When $f(x) = 2$ and the domain is $1 \le x < 2$, draw the graph $y = 2$ between the points (1, 2) and (2, 2).

When $f(x) = 1$ and the domain is $2 \le x < 3$, draw the graph $y = 1$ between the points (2, 1) and (3, 1).




Since the function cannot have two $y$ values for an $x$ value (otherwise, it would not be a function), you must look at the inequalities in the domain. When the inequality is less than or equal to ($\le$), you must draw the endpoint as a filled in circle. This shows that the function includes that point. For the strict inequalities ($<$), you must draw an endpoint with an open (not filled in) circle.

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**Example 6:** $f(x) = \begin{cases} x^2 & \text{if } x \geq 2 \\ 3 - x & \text{if } x < 2 \end{cases}$

Find a) $f(1)$
b) $f(3)$
c) $f(2)$

**Step 1:** Determine which interval of the domain includes the value of $x$.

a) For $f(1)$, $x = 1$. Since 1 is less than 2, you would plug $x = 1$ into $3 - x$.

b) For $f(3)$, $x = 3$. Since 3 is greater than 2, you would plug $x = 3$ into $x^2$.

c) For $f(2)$, $x = 2$. Since 2 is equal to 2, you would plug $x = 2$ into $x^2$.

**Step 2:** Plug the value of $x$ into the appropriate formula to solve for the value of $f(x)$.

a) $f(x) = 3 - x$, so $f(1) = 3 - 1 = 2$

b) $f(x) = x^2$, so $f(3) = (3)^2 = 9$

c) $f(x) = x^2$, so $f(2) = (2)^2 = 4$

**Graph each of the following functions. (DOK 2, 3)**

1. $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

2. $f(x) = \begin{cases} 1 & \text{if } x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$

3. $f(x) = \left\{ \begin{matrix} \sqrt{x} \text{ if } x \geq 2 \\ x^2 \text{ if } x < 2 \end{matrix} \right\}$

4. $f(x) = \begin{cases} 2x + 3 & \text{if } x < 0 \\ 2x - 3 & \text{if } x \geq 0 \end{cases}$

5. $f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ x & \text{if } -1 \leq x \leq 1 \\ -\left(x^2\right) & \text{if } x > 1 \end{cases}$

6. Phil's long distance phone service charges him 50 cents for the first 10 minutes and 10 cents for each minute afterwards. Graph the function that represents Phil's long distance phone service and find how much he would pay for

A) a 5 minute call.

B) a 10 minute call.

C) a 15 minute call.


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7. The tuition at State University is determined by the number of class hours a student takes. Tuition is \$100 for the first three hours and doubles every 3 hours. Graph the function that represents the tuition at State University and find the tuition for a student taking

   A) 6 class hours.

   B) 12 class hours.

   C) 15 class hours.

## 15.4 Solving Equations and Inequalities with Absolute Values

When solving equations and inequalities which invoive variables placed in absolute values, remember that there will be two or more numbers that will work as correct answers. This is because the absolute value variable will signify both positive and negative numbers as answers.

| | | |
|---|---|---|
| **Example 7:** | $5 + 3\|k\| = 8$ | Solve as you would any equation. |
| **Step 1:** | $3\|k\| = 3$ | Subtract 5 from each side. |
| **Step 2:** | $\|k\| = 1$ | Divide by 3 on each side. |
| **Step 3:** | $k = 1$ or $k = -1$ | Because $k$ is an absolute value, the answer can be 1 or –1. |
| **Example 8:** | $2\|x\| - 3 < 7$ | Solve as you normally would an inequality. |
| **Step 1:** | $2\|x\| < 10$ | Add 3 to both sides. |
| **Step 2:** | $\|x\| < 5$ | Divide by 2 on each side. |
| **Step 3:** | $x < 5$ or $x > -5$ or $-5 < x < 5$ | Because $x$ is an absolute value, the answer is a set of both positive and negative numbers. |

**Read each problem, and write the number or set of numbers which solves each equation or inequality.**

1. $7 + 2\|y\| = 15$
2. $4\|x\| - 9 < 3$
3. $6\|k\| + 2 = 14$
4. $10 - 4\|n\| > -14$
5. $-3 = 5\|z\| + 12$
6. $-4 + 7\|m\| < 10$
7. $5\|x\| - 12 > 13$
8. $21\|g\| + 7 = 49$
9. $-9 + 6\|x\| = 15$
10. $12 - 6\|w\| > -12$
11. $31 > 13 + 9\|r\|$
12. $-30 = 21 - 3\|t\|$
13. $9\|x\| - 19 < 35$
14. $-13\|c\| + 21 \geq -31$
15. $5 - 11\|k\| < -17$
16. $-42 + 14\|p\| = 14$
17. $15 < 3\|b\| + 6$
18. $9 + 5\|q\| = 29$
19. $-14\|y\| - 38 < -45$
20. $36 = 4\|s\| + 20$
21. $20 \leq -60 + 8\|e\|$

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## 15.5 More Solving Equations and Inequalities with Absolute Values

Now, look at the following examples in which numbers and variables are added or subtracted within the absolute value symbols (||).

**Example 9:** $|3x - 5| = 10$ Remember an equation with absolute value symbols has two solutions.

**Step 1:** $3x - 5 = 10$ To find the first solution, remove the absolute value symbol and solve the equation.

$3x - 5 + 5 = 10 + 5$

$\frac{3x}{3} = \frac{15}{3}$

$x = 5$

**Step 2:** $-(3x - 5) = 10$ To find the second solution, solve the equation for the negative of the expression in absolute value symbols.

$-3x + 5 = 10$

$-3x + 5 - 5 = 10 - 5$

$-3x = 5$

$x = -\frac{5}{3}$

**Solution:** $x = \{5, -\frac{5}{3}\}$

**Example 10:** $|5z - 10| < 20$ Remove the absolute value symbols and solve the inequality.

**Step 1:** $5z - 10 < 20$

$5z - 10 + 10 < 20 + 10$

$\frac{5z}{5} < \frac{30}{5}$

$z < 6$

**Step 2:** $-(5z - 10) < 20$ Next, solve the equation for the negative of the expression in the absolute value symbols.

$-5z + 10 < 20$

$-5z + 10 - 10 < 20 - 10$

$\frac{-5z}{5} < \frac{10}{5}$

$-z < 2$

$z > -2\}$

**Solution:** $-2 < z < 6$


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**Example 11:** $|4y + 7| - 5 > 18$

**Step 1:**
$4y + 7 - 5 + 5 > 18 + 5$
$4y + 7 > 23$
$4y + 7 - 7 > 23 - 7$
$4y > 16$
$y > 4$

Remove the absolute value symbols and solve the inequality.

**Step 2:**
$-(4y + 7) - 5 > 18$
$-4y - 7 - 5 + 5 > 18 + 5$
$-4y - 7 + 7 > 23 + 7$
$-4y > 30$
$y < -7\frac{1}{2}$

Solve the inequality for the negative of the expression in the absolute value symbols.

**Solution:** $y > |4$ or $y < -7\frac{1}{2}$

**Solve the following equations and inequalities below.**

1. $-4 + |2x + 4| = 14$
2. $|4b - 7| + 3 > 12$
3. $6 + |12e + 3| < 39$
4. $-15 + |8f - 14| > 35$
5. $|-9b + 13| - 12 = 10$
6. $-25 + |7b + 11| < 35$
7. $|7w + 2| - 60 > 30$
8. $63 + |3d - 12| = 21$
9. $|-23 + 8x| - 12 > 37$
10. $|61 + 20x| + 32 > 51$
11. $|4a + 13| + 31 = 50$
12. $4 + |4k - 32| < 51$
13. $8 + |4x + 3| = 21$
14. $|28 + 7v| - 28 < 77$
15. $|62p + 31| + 43 = 136$
16. $18 - |6v + 22| < 22$
17. $12 = 4 + |42 + 10m|$
18. $53 < 18 + |12e + 31|$
19. $38 > -39 + |7j + 14|$
20. $9 = |14 + 15u| + 7$
21. $11 - |2j + 50| > 45$
22. $|35 + 6i| - 3 = 14$
23. $|26 - 8r| - 9 > 41$
24. $|25 + 6z| - 21 = 28$
25. $12 < |2t + 6| - 14$
26. $50 > |9q - 10| + 6$
27. $12 + |8v - 18| > 26$
28. $-38 + |16i - 33| = 41$
29. $|-14 + 6p| - 9 < 7$
30. $28 > |25 - 5f| - 12$

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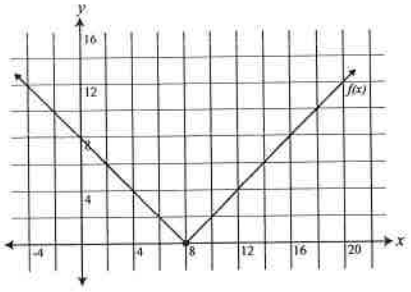
## 15.6 Solving Absolute Value Equations Graphically

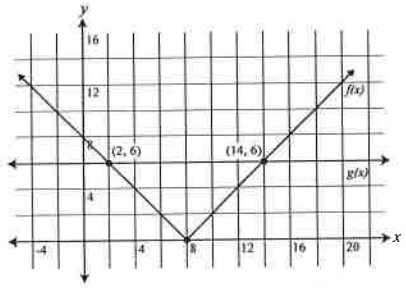
To solve an absolute value equation graphically, consider one side of the equation as one function and the other side of the equation as another function. Then graph the two functions and see where they intersect.

**Example 12:** Solve the absolute value equation $|x - 8| = 6$ graphically.

**Step 1:** First, rewrite the absolute value part of the function as $f(x) = |x - 8|$. Graph the function.




**Step 2:** Now, rewrite the other side of the equation as $g(x) = 6$. Graph the function on the same graph.




The solutions are the $x$-values when the $y$-value is 6. Looking at the graph, we can see that the two functions intersect at the points (2, 6) and (14, 6), so the solutions to the equation are $x = 2$ or $x = 14$.

**Graph each of the following absolute value equations to find the solution, then check your answer analytically.**

1. $|x + 4| = 11$
2. $|x - 3| = 8$
3. $|x - 5| = -1$
4. $|x + 10| = 2$
5. $|x - 7| = 0$
6. $|x + 6| = 13$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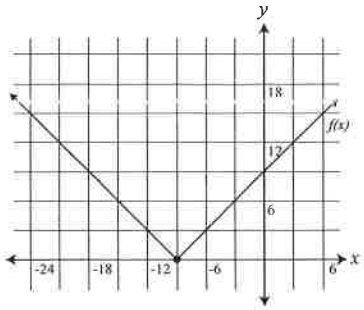
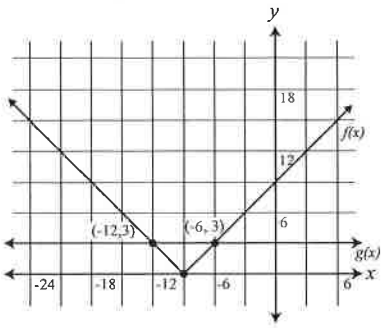
**Example 13:** Solve the absolute value equation $-5|x + 9| + 2 = -13$ graphically.

**Step 1:** First, get the absolute value part of the equation to one side of the equation by itself.
Subtract 2 from both sides of the equation so that it becomes $-5|x + 9| = -15$, and then divide both sides of the equation by $-5$ so that it becomes $|x + 9| = 3$.

**Step 2:** Now, rewrite the absolute value part of the function as $f(x) = |x + 9|$. Graph the function.




**Step 3:** Now, rewrite the other side of the equation as $g(x) = 3$. Graph the function on the same graph.




The solutions are the $x$-values when the $y$-value is 3. Looking at the graph, we can see that the two functions intersect at the points (12, 3) and (6, 3), so the solutions to the equation are $x = 12$ or $x = -6$.

**Graph each of the following absolute value equations to find the solution, then check your answer analytically.**

1. $6|x + 2| - 1 = 11$
2. $-3|x - 8| + 4 = 17$
3. $\frac{3}{5}|x + 10| = 24$
4. $7|x - 1| - 5 = 9$
5. $-2|x| + 17 = 19$
6. $8|x + 7| - 2 = 46$

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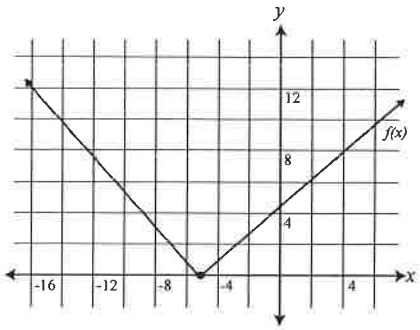
## 15.7 Solving Absolute Value Inequalities Graphically

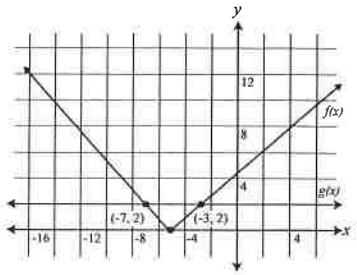
To solve an absolute value inequality graphically, consider one side of the inequality as one function and the other side of the inequality as another function. Then graph the two functions and see where they meet the conditions of the inequality.

**Example 14:** Solve the absolute value inequality $|x + 5| < 2$ graphically.

**Step 1:** First, rewrite the absolute value part of the function as $f(x) = |x + 5|$. Graph the function.




**Step 2:** Now, rewrite the other side of the inequality as $g(x) = 2$. Graph the function on the same graph.




A dotted line is graphed at $g(x) = 2$. All of the $x$-values on the graph $f(x) = |x + 5|$ that are below (or less than) the graph of $g(x) = 2$ are solutions. The solution to the inequality is $-7 < x < -3$.

**Graph each of the following absolute value inequalities to find the solution, then check your answer analytically.**

1. $|x - 7| < 8$
2. $|x + 11| \leq 3$
3. $|x + 9| < 16$
4. $|x - 2| < 21$
5. $|x - 12| \leq 1$
6. $|x + 4| < 17$



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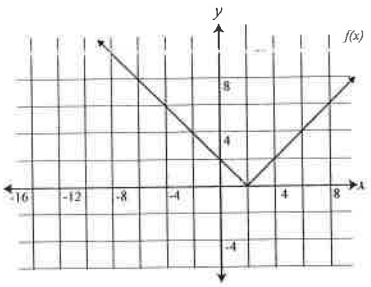
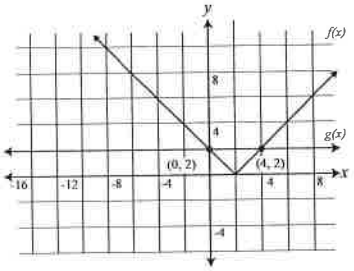
**Example 15:** Solve the absolute value inequality $7|x-2|-8 \geq 6$ graphically.

**Step 1:** First, get the absolute value part of the inequality to one side of the equation by itself.
First add 8 to both sides of the inequality $7|x-2|-8 \geq 6$ so that it becomes $7|x-2| \geq 195$, and then to divide both sides of the inequality by 7 so that it becomes $|x-2| \geq 2$.

**Step 2:** Now, rewrite the absolute value part of the function as $f(x) = |x-2|$. Graph the function.




**Step 3:** Now, rewrite the other side of the inequality as $g(x) = 2$. Graph the function on the same graph.




All of the $x$-values on the graph $f(x) = |x-2|$ that are above or on (greater than or equal to) the graph of $g(x) = 2$ are solutions. The solution to the inequality is $x \leq 0$ or $x \geq 4$.

**Graph each of the following absolute value inequalities to find the solution, then check your answer analytically.**

1. $3|x+9| > 24$
2. $-4|x-10|+5 \geq -11$
3. $2|x-6|-13 > 1$
4. $8|x+5|+4 \geq 36$
5. $9|x+1|-2 \geq 43$
6. $5|x-18|-6 > 9$

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## Chapter 15 Review

**Solve the following equations and inequalities below.**

1. $-11|k| < -22$
2. $18 - 3|w| > -18$
3. $2|2(6x + 1)| = 6$
4. $21 = -4 + |5x + 5|\,|$
5. $|x + \frac{7}{10}| = 1$
6. $|\frac{7x}{2} + -7| = 4$
7. $|3x + 2| - 4 \geq -2$
8. $|10x + 4\,| < 7$
9. $|-3x\ \ 7| \geq 11$
10. $4|-6x + 8| > 12$
11. $|5(3x - 1)| = 20$
12. $|\frac{x}{6} - 16| \leq 5$
13. $|6x + 1| = 11$
14. $7|4x + 5| \geq 35$
15. $|15x - 2| > \frac{3}{4}$
16. $|4x - 3| = 10$
17. $|-2x + 9| = 13$
18. $|\frac{2x}{5} - 8| = 9$

**Solve each of the following absolute value equations graphically.**

19. $|x + 16| = 2$
20. $|x - 9| = 5$
21. $4|x - 3| = 52$
22. $-7|x + 5| + 2 = -12$
23. $\frac{1}{18}|3(x + 5)| + 6 = 9$
24. $8|2(x - 1)| - 1 = 15$

**Solve each of the following absolute value inequalities graphically.**

25. $|x - 9| \leq 22$
26. $|x + 17| > 31$
27. $3|x - 7| - 19 < 5$
28. $4|x + 13| - 24 > 8$
29. $8|5(x - 7)| - 29 < 11$
30. $7|2(x + 8)| + 3 > 59$



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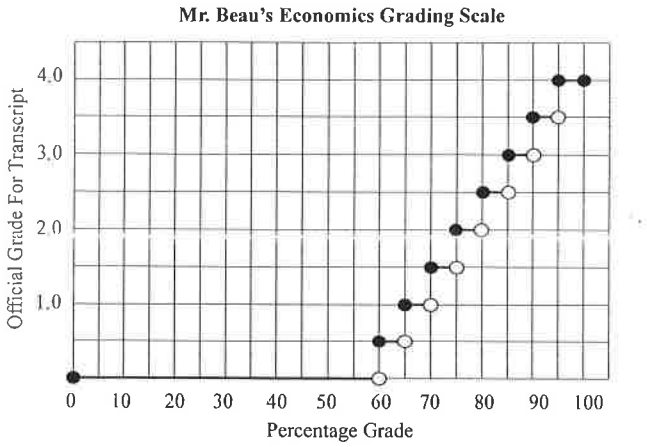
**Answer the following questions about the nonlinear graphs. (DOK 2)**

Notre Pere College Preparatory Academy uses a 4.0 scale for its grades, to emphasize its college preparatory curriculum. Mr. Beau teaches the senior economics class, and he uses the grading scale graphed below.

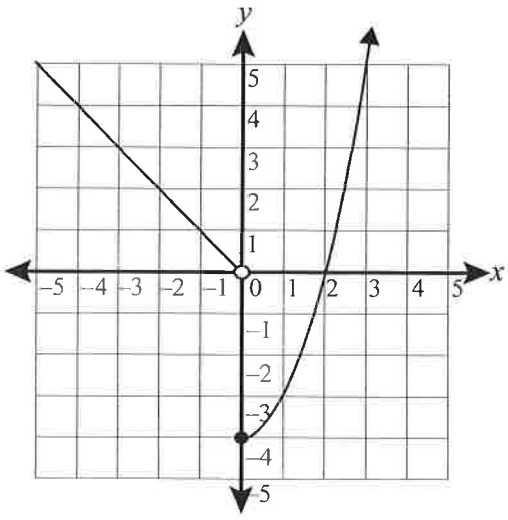



31. MaryLee scored an average of 83% in the course. What was her official 4.0 scale grade?

32. What are the domain and range of this function?

**Answer questions 33–34 about the graph. (DOK 2)**

The graph of function $y = \begin{cases} -x \text{ where } x < 0 \\ x^2 - 4 \text{ where } x \geq 0 \end{cases}$ is shown below.




33. Where is the graph discontinuous?

34. What is (are) the zero(s) of this graph?

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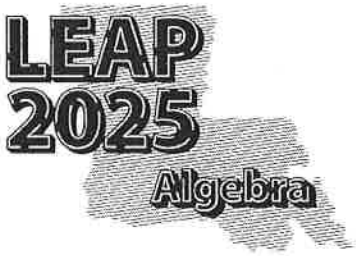
# Chapter 16
# Patterns




Standards covered: F.LE.A.2, F.IF.A.3, F.BF.A1a, F.BF.2

## 16.1 Number Patterns (DOK 2)

In each of the examples below, there is a sequence of numbers that follows a pattern. Think of the sequence of numbers like the output for a function. You must find the pattern (or function) that holds true for each number in the sequence. Once you determine the pattern, you can find the next number in the sequence or any number in the sequence.

| | Sequence | Pattern | Next Number | 20th number in the sequence |
|---|---|---|---|---|
| **Example 1:** | 3, 4, 5, 6, 7… | $n + 2$ | 8 | 22 |

In number patterns, the sequence is the output. The input can be the set of whole numbers starting with 1. But, you must determine the "rule" or pattern. Look at the table below.

| input | sequence |
|---|---|
| 1 | → 3 |
| 2 | → 4 |
| 3 | → 5 |
| 4 | → 6 |
| 5 | → 7 |

What pattern or "rule" can you come up with that gives you the first number in the sequence, 3, when you input 1? $n + 2$ will work because when $n = 1$, the first number in the sequence = 3. Does this pattern hold true for the rest of the numbers in the sequence? When $n = 2$, the second number in the sequence equals 4. When $n = 3$, the third number in the sequence equals 5, and so on. Therefore, $n + 2$ is the pattern. Even without knowing the algebraic form of the pattern, you could determine that 8 is the next pattern in the sequence. To find the 20th number in the pattern, use $n = 20$ to get 22.

| | Sequence | Pattern | Next Number | 20th Number in the Sequence |
|---|---|---|---|---|
| **Example 2:** | 1, 4, 9, 16, 25… | $n^2$ | 36 | 400 |
| **Example 3:** | −2, −4, −6, −8, −10… | $-2n$ | −12 | −40 |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**Find the pattern, the next number, and the 20th number in each of the sequences below. (DOK 2)**

| | Sequence | Pattern | Next Number | 20th Number in the Sequence |
|---|---|---|---|---|
| 1. | −2, −1, 0, 1, 2… | _____ | _____ | _____ |
| 2. | 3, 7, 11, 15, 19… | _____ | _____ | _____ |
| 3. | −3, −6, −9, −12, −15… | _____ | _____ | _____ |
| 4. | 1, 8, 27, 64, 125… | _____ | _____ | _____ |
| 5. | 0, −1, −2, −3, −4… | _____ | _____ | _____ |
| 6. | 4, 6, 8, 10, 12… | _____ | _____ | _____ |

## 16.2 Identifying Arithmetic and Geometric Series (DOK 1)

A **series** is the sum of a sequence. Two examples of series are arithmetic and geometric series. An **arithmetic series** is a series where the difference between each successive term is a fixed constant. A **geometric series** is a series where each term is being multiplied by the same value to get the next term.

**Example 1:** Identify the following series as arithmetic or geometric:
$3 + 4 + 5 + 6 + 7 + \ldots$

**Step 1:** We will first check to see if this is an arithmetic series. If this is an arithmetic series, then the difference between a term in the series and the term before it should always be the same number.
$4 - 3 = 1$
$5 - 4 = 1$
$6 - 5 = 1$
$7 - 6 = 1$

**Step 2:** Since the difference between a term and the term immediately before it is always 1, then this series is **arithmetic**.

**Example 2:** Identify the following series as arithmetic or geometric:
$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \ldots$

**Step 1:** We will first check to see if this is an arithmetic series. If this is an arithmetic series, then the difference between a term in the series and the term before it should always be the same number.
$\frac{1}{9} - \frac{1}{3} = -\frac{2}{9}$
$\frac{1}{27} - \frac{1}{9} = -\frac{2}{27}$

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**Step 2:** Since the difference between terms is not the same value, we know that this is not an arithmetic series, so we must check to see if it is a geometric series. If this is a geometric series, then each term is multiplied by a constant number to get the next term.

$\frac{1}{9} \div \frac{1}{3} = \frac{1}{3}$

$\frac{1}{27} \div \frac{1}{9} = \frac{1}{3}$

$\frac{1}{81} \div \frac{1}{27} = \frac{1}{3}$

$\frac{1}{243} \div \frac{1}{81} = \frac{1}{3}$

**Step 3:** Since each number is being multiplied by $\frac{1}{3}$ to get the next number in the sequence, this series is **geometric**.

**Identify the series as arithmetic or geometric. (DOK 1)**

1. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$
2. $5 + 6 + 7 + 8 + 9 + \ldots$
3. $0 + -1 + -2 + -3 + -4 + \ldots$
4. $32 + 8 + 2 + \frac{1}{2} + \frac{1}{8} + \ldots$
5. $-1 + 0 + 1 + 2 + 3 + \ldots$
6. $-625 + -125 + -25 + -5 + \ldots$

## 16.3 Arithmetic Series (DOK 2)

An **arithmetic sequence** is an ordered list of numbers where the difference between each successive number is constant. The sequence {1, 2, 3, 4, ...} is an arithmetic sequence because the difference between one member and the next is 1. An **arithmetic series** is the sum of an arithmetic sequence of numbers.

To find any term of an arithmetic series, the formula $a_n = a_1 + d(n - 1)$ can be used, where $a_1$ is the first term, $d$ is the common difference, and $n$ is the number of the term to find.

To calculate the sum of the first $n$ terms of an arithmetic series, use the formula $S_n = \frac{n(a_1 + a_n)}{2}$, where $a_1$ is the first term in the series and $a_n$ is the $n$th term in the series. This formula will not be provided on your formula sheet.

**Example 1:** Find the sum of the first 30 terms of the arithmetic series:
$3 + 4 + 5 + 6 + 7 + \ldots$

**Step 1:** First, find the pattern of the series. The pattern of this series is $n + 2$.


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**Step 2:** Next find all the variables for the summing equation, $S_n = \frac{n(a_1 + a_n)}{2}$.
$a_1 = 3$
$n = 30$ (how many terms we will find the sum of)
$a_n = a_{30} = 30 + 2 = 32$

**Step 3:** Plug all the values into the equation. $S_{30} = \frac{30(3+32)}{2} = 525$.
The sum of the arithmetic series is 525.

**Example 2:** Calculate the sum of the first 10 natural numbers.

**Method 1:** We can manually write out and compute the sum:.
$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$

**Method 2:** We can use the formula to find $S_{10,}$ where $n = 10$, $a_1 = 1$, and $a_{10} = 10$.
$S_{10} = \frac{n(a_1 + a_{10})}{2} = \frac{10(1+10)}{2} = 55$

**Example 3:** What is the sum of first six terms in the arithmetic series $2 + 5(n - 1)$?

**Step 1:** Write out the numbers in the arithmetic series.
The first number, $n = 1$, in the arithmetic series is
$2 + 5(1 - 1) = 2 + 5(0) = 2 + 0 = 2$, while the second number in the arithmetic series is $2 + 5(2 - 1) = 2 + 5(1) = 2 + 5 = 7$. For this reason, the common difference of the arithmetic series must be $7 - 2$, or 5, and since there are 6 terms in the arithmetic series, it must be $2 + 7 + 12 + 17 + 22 + 27$.

**Step 2:** Add the numbers in the arithmetic series together.
$2 + 7 + 12 + 17 + 22 + 27 = 87$

**Find the sum of the first 20 terms in the finite arithmetic series. (DOK 2)**

1. $-3 + -6 + -9 + -12 + -15 + \ldots$
2. $0 + -1 + -2 + -3 + -4 + \ldots$
3. $4 + 6 + 8 + 10 + 12 + \ldots$
4. $-2 + -1 + 0 + 1 + 2 + \ldots$

**Calculate the sum of the arithmetic series with each of the following descriptions. (DOK 2)**

5. $a_1 = 12, a_{16} = 117, n = 16$
6. $a_1 = -13, a_{22} = 92; n = 22$
7. $a_1 = 150, a_{25} = -354, n = 25$
8. $a_1 = 7, a_{51} = -293, n = 51$

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**Calculate the sum of each of the following groups of numbers. (DOK 2)**

9. the first 23 positive even integers

10. the first 31 multiples of 5

**Calculate the specified sum of each of the following arithmetic series. (DOK 2)**

11. the first 6 terms of $6 - 2(n - 1)$

12. the first 5 terms of $-11 + 5(n - 1)$

**Find the 12th term in each of these series using the formula $a_n = a_1 + d(n - 1)$. (DOK 2)**

13. $5 + 6 + 7 + 8 + 9 + \ldots$

14. $a_1 = -13$, $a_{22} = 92$; $n = 22$

## 16.4 Arithmetic Sequences (DOK 2)

If you list the partial sums of an arithmetic series repeatedly, you will get a sequence. This sequence can often be described using quadratic functions.

The rate of change of a quadratic function changes at a constant rate. For example, four points on the graph of the function $f(x) = x^2$ are (1, 1), (2, 4), (3, 9), and (4, 16).

Taking the differences between each $f(x)$ value and the previous $f(x)$ value, we get $4 - 1 = 3$, $9 - 4 = 5$, and $16 - 9 = 7$.

The first change in $f(x)$ is 3 units, the second change is 5 units, and the third change is 7 units. For this reason, for every change of 1 unit of $x$, the change in $f(x)$ is increasing by 2 units each time.

The sequences of partial sums of arithmetic series work in the same way, and are therefore examples of quadratic functions.

**Example 1:** Calculate the first differences and the second differences of the sequence of partial sums of the arithmetic series $3 + 8 + 13 + 18 + 23 + 28 + 33$.

**Step 1:** Determine the sequence of partial sums of the arithmetic series.
The partial sums of the arithmetic series are as follows:
$S_1 = 3$
$S_2 = 3 + 8 = 11$
$S_3 = 3 + 8 + 13 = 24$
$S_4 = 3 + 8 + 13 + 18 = 42$
$S_5 = 3 + 8 + 13 + 18 + 23 = 65$
$S_6 = 3 + 8 + 13 + 18 + 23 + 28 = 93$
$S_7 = 3 + 8 + 13 + 18 + 23 + 28 + 33 = 126$

**Step 2:** Calculate the first differences of the sequence of partial sums of the arithmetic series.


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The sequence of partial sums of the arithmetic series is 3, 11, 24, 42, 65, 93, 126. The first differences of the sequence can be calculated by subtracting each partial sum from the partial sum that follows it.

$11 - 3 = 8$ $\quad$ $42 - 24 = 18$ $\quad$ $93 - 65 = 28$

$24 - 11 = 13$ $\quad$ $65 - 42 = 23$ $\quad$ $126 - 93 = 33$

**Step 3:** Calculate the second differences of the sequence of partial sums of the arithmetic series.

The sequence of first differences of the partial sums of the arithmetic series is 8, 13, 18, 23, 28, 33. The second differences of the sequence can be calculated by subtracting each first difference from the first difference that follows it.

$13 - 8 = 5$ $\quad$ $23 - 18 = 5$ $\quad$ $33 - 28 = 5$

$18 - 13 = 5$ $\quad$ $28 - 23 = 5$

The second differences are all 5. When the second differences are all the same, it tells us that the polynomial for this sequence of the partial sums of an arithmetic series of values is a quadratic function.

**Calculate the first differences and the second differences of the sequence of partial sums of each of the following arithmetic series. (DOK 2)**

1. $6 + 9 + 12 + 15 + 18$

2. $-8 + 0 + 8 + 16 + 24$

3. $9 + 7 + 5 + 3 + 1$

4. $54 + 42 + 30 + 18 + 6$

**Example 2:** The sequence of first differences of the partial sums of a series is 2, 4, 8, 16, 32. Determine if the series is arithmetic.

**Step 1:** Calculate the second differences of the sequence of partial sums of the series. The second differences of the sequence can be calculated by subtracting each first difference from the first difference that follows it. Therefore, the second differences of the sequence are as follows:

$4 - 2 = 2$ $\quad$ $16 - 8 = 8$

$8 - 4 = 4$ $\quad$ $32 - 16 = 16$

**Step 2:** Determine if the second differences are all the same. If so, the series is arithmetic. If not, the series is not arithmetic.

The second differences are not all the same. Therefore, the series is not arithmetic.

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**Determine if the series with each of the following sequences of first differences of partial sums is arithmetic. (DOK 2)**

5. 11, 19, 27, 35, 43

6. 2, 10, 50, 250, 1250

## 16.5 Geometric Sequences (DOK 2)

A **geometric sequence** is a pattern where each term is multiplied by the same value to get the next term. The formula for finding the $n$th term in a geometric sequence is $t_n = t_1 r^{n-1}$, where $t_1$ is the first term in the series, and $t_n$ is the $n$th term in the series. In the geometric series, $r$ is the value multiplied by each term to reach the next term in the series. This means that $r$ is a constant ratio in the geometric sequence. Therefore, geometric sequences are examples of exponential functions.

**Example 1:** Below is a geometric sequence. Find the formula for the sequence. Then find the 8th term in the sequence.

$27, 9, 3, 1, \frac{1}{3}, \ldots$

**Step 1:** The formula $tn = t_1 r^{n-1}$ used to find the $n$th term in a geometric sequence is the same formula we will use to find the formula for the pattern. First, we need to find the missing variables $t_1$ and $r$. $t_1$ is the first term of the sequence, 27.

**Step 2:** The variable $r$ is the constant in the sequence. This is the number that each term is multiplied by to find the next term.

$9 \div 27 = \frac{1}{3}$ $\qquad$ $1 \div 3 = \frac{1}{3}$

$3 \div 9 = \frac{1}{3}$ $\qquad$ $\frac{1}{3} \div 1 = \frac{1}{3}$

**Step 3:** Since each number is being multiplied by $\frac{1}{3}$ to get the next number in the sequence, $r = \frac{1}{3}$.

Thus, the formula for the sequence is $t_n = 27\left(\frac{1}{3}\right)^{n-1}$.

**Step 4:** To find the 8th term in the sequence, substitute 8 into the formula for $n$.

$$t_8 = 27\left(\frac{1}{3}\right)^{8-1} = \frac{1}{81}$$

**Find the formula for the sequences. Then find the 10th term in the sequence. (DOK 2)**

1. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

2. $100, 10, 1, \frac{1}{10}, \frac{1}{100}, \ldots$

3. $-625, -125, -25, -5, \ldots$

4. $-9, -3, -1, -\frac{1}{3}, -\frac{1}{9}, \ldots$


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## 16.6 Geometric Series (DOK 2)

As you know, a series is the sum of a sequence. You learned about one kind, finite arithmetic series, in a previous section. There are also finite and infinite geometric series. A geometric series is a series where each term is being multiplied by the same value to get the next term. There are specific formulas for the sum of each type of series.

| Series | Formula for the Sum |
|---|---|
| Finite Geometric Series | $S_n = \frac{t_1(1-r^n)}{(1-r)}$ |
| Infinite Geometric Series | $S = \frac{t_1}{1-r}$ |

The sum is denoted by $S$ ($S_n$ is the sum of the series to a specific term, $n$). The first term in the series $t_1$ is the first term in the series, and $t_n$ is the $n$th term in the series. In the geometric series, $r$ is the value being multiplied to each term to reach the next term in the series.

**NOTE:** In the infinite geometric series, if $|r| < 1$, then the sum of the series can be found. If $|r| > 1$, then the sum of the series cannot be found.

**Example 1:** Find the sum of the first 5 terms of the finite geometric series:
$\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\frac{1}{243}$

**Step 1:** Find the value of $r$, by which each term is multiplied to get the next number in the series. $r$ in this series is $\frac{1}{3}$.

**Step 2:** Find all the variables for the summing equation, $S_n = \frac{t_1(1-r^n)}{(1-r)}$

$t_1 = \frac{1}{3}$

$n = 5$

$r = \frac{1}{3}$

**Step 3:** Insert the values in the equations: $S_5 = \frac{\frac{1}{3}\left(1-\left(\frac{1}{3}\right)^5\right)}{\left(1-\frac{1}{3}\right)} = \frac{\frac{1}{3}\left(\frac{242}{243}\right)}{\left(\frac{2}{3}\right)} = \frac{121}{243}$

The sum of the geometric series is $\frac{121}{243} = 0.498$.

**Example 2:** Find the sum of the infinite geometric series:
$16 + 8 + 4 + 2 + 1 + \ldots$

**Step 1:** Find the value, $r$, by which each term is multiplied to get the next number in the series. In this series, $r$ is $\frac{1}{2}$.

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**Step 2:** Find all the values for the equation, $S = \frac{t_1}{1-r}$.

**Step 3:** Determine if $|r| < 1$ or if $|r| > 1$.
$|r| = \left|\frac{1}{2}\right| = \frac{1}{2} < 1$. Therefore, there is a sum for this series.

**Step 4:** Insert the values into the summing equation: $S = \frac{16}{1-\frac{1}{2}} = 32$.

The sum of the infinite geometric series is 32.

**Find the sum of the first 10 terms in the finite geometric series. (DOK 2)**

1. $25 + 5 + 1 + \frac{1}{5} + \ldots$

2. $100 + 10 + 1 + \frac{1}{10} + \frac{1}{100} + \ldots$

3. $32 + 8 + 2 + \frac{1}{2} + \frac{1}{8} + \ldots$

4. $-9 + -3 + -1 + -\frac{1}{3} + -\frac{1}{9} + \ldots$

**Find the sum of the infinite geometric series. Remember the sum can only be found if $|r| < 1$. If the sum cannot be found, write NP. (DOK 2)**

5. $-625 + -125 + -25 + -5 + \ldots$

6. $2 + 4 + 8 + 16 + 32 + \ldots$

7. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$

8. $3 + 9 + 27 + 81 + 243 + \ldots$

## 16.7 Explicit and Recursive Functions (DOK 2)

Explicit functions generate a pattern and do not need to reference any previous numbers in order to do so. Recursive functions also generate a pattern, but they do so by referencing the previous term and applying it into the function. Both explicit and recursive functions use the notation $a_n$, in which $n$ stands for the number of the step in the pattern.

**Example 1:** $a_n = 3^n$ This is an **explicit** function.

**Step 1:** $a_1 = 3^1$ Substitute 1 for $n$ because we are in step 1.

Thus, $a_1 = 3$.


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**Step 2:** $a_2 = 3^2$ Substitute 2 for $n$ because we are in step 2.

Thus, $a_2 = 9$.

**Step 3:** $a_3 = 3^3$ Substitute 3 for $n$ because we are in step 3.

Thus, $a_3 = 27$.

**Step 4:** $a_4 = 3^4$ Substitute 4 for $n$ because we are in step 4.

Thus, $a_4 = 81$.

**Example 2:** $a_n = 3_{(n-1)}$ where $a_0 = 1$ This is a **recursive** function.

**Step 1:** $a_1 = 3_{a(1-1)}$ Substitute 1 for $n$ because we are in step 1.

$a_1 = 3_{a(0)}$ $1 - 1 = 0$

$a_1 = 3 \times 1$ $a_{(0)} = 1$ is given, so substitute the number 1 into formula for $a_{(0)}$ then multiply.

Thus, $a_1 = 3$.

**Step 2:** $a_2 = 3a_{(2-1)}$ Substitute 2 for $n$ because we are in step 2.

$a_2 = 3a_{(1)}$ $2 - 1 = 1$

$a_2 = 3 \times 3$ $a_{(1)} = 3$ was calculated in the previous step, so substitute 3 into formula for $a_{(1)}$ then multiply.

Thus, $a_2 = 9$.

**Step 3:** $a_3 = 3a_{(3-1)}$ Substitute 3 for $n$ because we are in step 3.

$a_3 = 3a_{(2)}$ $3 - 1 = 2$

$a_3 = 3 \times 9$ $a_{(2)} = 9$ was calculated in the previous step, so substitute 9 into formula for $a_{(2)}$ then multiply.

Thus, $a_3 = 27$.

**Step 4:** $a_4 = 3a_{(4-1)}$ Substitute 4 for $n$ because we are in step 4.

$a_4 = 3a_{(3)}$ $4 - 1 = 3$

$a_4 = 3 \times 27$ $a_{(3)} = 27$ was calculated in the previous step, so substitute 27 into formula for $a_{(3)}$, then multiply.

Thus, $a_4 = 81$.

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Another recursive function is known as the Fibonacci sequence. Leonardo Fibonacci was an Italian mathematician. He is most known for the number sequence named after him in which each term is the sum of the two previous terms.

**Example 3:** The Fibonacci sequence is a list of numbers, for which the $n^{th}$ term is the sum of the two previous terms. It can be defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

Therefore, $f(0) = 1, f(1) = 1, f(2) = 2, f(3) = 3, f(4) = 5, f(5) = 8, f(6) = 13$, and so on.

**Define the following sequences as a function and find the next term in the sequence. (DOK 2)**

1. 1, 3, 5, 7, 9 …
2. 1, 2, 4, 8, 16, 32 …
3. 1, 4, 9, 16, 25, 36 …
4. 5, 10, 15, 20, 25 …

**Example 4:** Consider the sequence 3, 9, 27, 81 …
This sequence can be defined by a recursive or an explicit formula. An explicit formula describes a term without reference to other numbers in the sequence, while a recursive formula defines a term using other terms in the sequence.

**Answer:** Explicit formula: $f(n) = 3n, n \geq 1$
Recursive formula: $f(1) = 3, f(n) = 3f(n-1)$

**Write the following sequences in explicit form and recursive form. (DOK 2)**

5. 2, 4, 6, 8, 10 …
6. 9, 18, 27, 36, 54 …
7. 19, 27, 35, 43 …
8. 16, 24, 32, 40 …


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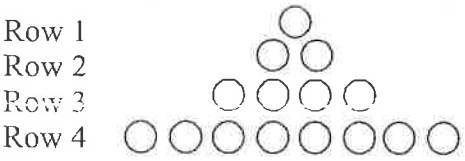
## 16.8 Solving Real-World Patterns (DOK 3)

Humans have always observed what happened in the past and used these observations to predict what would happen in the future. This is called **inductive reasoning**. We observe patterns in the mathematical behavior of a phenomenon, then find a rule or formula for describing and predicting its future mathematical behavior.

**Example 1:** Nancy is watching her nephew, Drew, arrange his marbles in rows on the kitchen floor. The figure below shows the progression of his arrangement.




**Assuming this pattern continues, how many marbles would Drew place in a fifth row?**

**Solution:** It appears that Drew doubles the number of marbles in each successive row. In the 4th row he had 8 marbles, so in the 5th row we can predict 16 marbles.

**Example 2:** Manuel drops a golf ball from the roof of his high school while Carla video tapes the motion of the ball. Later, the video is analyzed and the results are recorded concerning the height of each bounce of the ball.

**What height do you predict for the fifth bounce?**

| Initial height | 1st bounce | 2nd bounce | 3rd bounce | 4th bounce |
|---|---|---|---|---|
| 30 ft | 18 ft | 10.8 ft | 6.48 ft | 3.888 ft |

To answer this question, we need to be able to relate the height of each bounce to the bounce immediately preceding it. Perhaps the best way to do this is with ratios as follows:

$$\frac{\text{Height of 1st bounce}}{\text{Initial bounce}} = 0.6 \qquad \frac{\text{Height of 2nd bounce}}{\text{Height of 1st bounce}} = 0.6$$

$$\frac{\text{Height of 4th bounce}}{\text{Height of 3rd bounce}} = 0.6$$

Since the ratio of the height of each bounce to the bounce before it appears constant, we have some basis for making predictions.

Using this, we can reason that the fifth bounce will be equal to 0.6 of the fourth bounce.

**Solution:** Thus we predict the fifth bounce to have a height of $0.6 \times 3.888 = 2.3328$ ft.

**Which bounce will be the last one with a height of one foot or greater?**

For this question, keep looking at predicted bounce heights until a bounce of less than 1 foot is reached.

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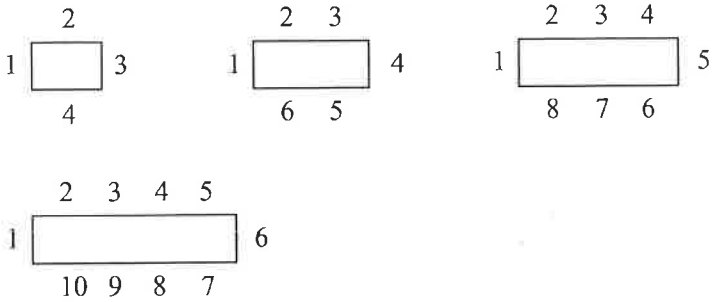
The sixth bounce is predicted to be $0.6 \times 2.3328 = 1.399768$ ft.

The seventh bounce is predicted to be $0.6 \times 1.399768 = 0.839808$ ft.

**Solution:** Thus, the last bounce with a height greater than 1 ft is predicted to be the sixth one.

**Example 3:** Mr. Applegate wants to put desks together in his math class so that students can work in groups. The diagram below shows how he wishes to do it.




With one table he can seat 4 students, with two tables he can seat 6, with three tables 8, and with four tables 10.

**How many students can he seat with 5 tables?**

**Solution:** With 5 tables he could seat 5 students along the sides of the tables and 1 student on each end; thus, a total of 12 students could be seated.

**Write a rule that Mr. Applegate could use to tell how many students could be seated at *n* tables. Explain how you got the rule.**

**Solution:** For $n$ tables, there would be $n$ students along each of 2 sides and 2 students on the ends (1 on each end); thus, a total of $2n + 2$ students could be seated at $n$ tables.

**Example 4:** When he isn't playing football for the Burlington Bears, Tim designs web pages. A car dealership paid Tim \$500 to start a site with photos of its cars. The dealer also agreed to pay Tim \$50 for each customer who buys a car first viewed on the website.

**Write and explain a rule that tells how much the dealership will pay Tim for the design of the website and the sale of *n* cars from the website.**

**Solution:** Tim's payment will be the initial \$500 plus \$50 for each sale. Translated into mathematical language, if Tim sells $n$ cars he will be paid a total of $500 + 50n$ dollars.

**How many cars have to be sold from his site in order for Tim to get \$1000 from the dealership?**

**Solution:** He earned \$500 just by establishing the site, so he only needs to earn an additional \$500, which at \$50 per car requires the sale of only 10 cars. (Note: Another way to solve this problem is to use the rule found in the first question. In that case, you simply solve the equation $500 + 50n = 1000$ for the variable $n$.)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**Read the following questions carefully. Use inductive reasoning to answer each question. You may wish to make a table or a diagram to help you visualize the pattern in some of the problems. (DOK 3)**

1. Bob and Alice have designed and created a website for their high school. The first week they had 5 visitors to the site; the second week, they had 10 visitors; and during the third week, they had 20 visitors.

   (A) If current trends continue, how many visitors can they expect in the fifth week?

   (B) How many in the *n*th week?

   (C) How many weeks will it be before they get more than 500 visitors in a single week?

2. In 1979 (the first year of classes), there were 500 students at Brookstone High. In 1989, there were 1000 students. In 1999, there were 2000 students. How many students would you predict were at Brookstone in 2009 if this pattern continued (and no new schools are built)?

3. After a volcano erupted, scientists studied how far the volcanic ash spread over time. Every Saturday morning, they spread out to measure and records its farthest reach in the table below. What distance do you predict for ash to spread on May 2? (Hint: Look at the change in distance each week when looking for the pattern).

| April 4 | April 11 | April 18 | April 25 |
|---|---|---|---|
| 12 km | 18 km | 21 km | 22.5 km |

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## Chapter 16 Review

**Find the pattern for the following number sequences, and then find the *n*th number requested. (DOK 2)**

1. 0, 1, 2, 3, 4 pattern _____; 20th number _____

2. 1, 3, 5, 7, 9 pattern _____; 20th number _____

**Identify the series as arithmetic or geometric. (DOK 1)**

3. $0 + 1 + 2 + 3 + 4 + \ldots$

4. $-32 + -16 + -8 + -4 + -2 + \ldots$

**Calculate the specified sum of each of the following arithmetic series. (DOK 1)**

5. the first 8 terms of $10 - 3(n - 1)$

6. the first 7 terms of $14 + 4(n - 1)$

**Find the sum of the finite series when $n = 10$. (DOK1)**

7. $0 + -1 + -2 + -3 + -4 + \ldots$

8. $2 + 5 + 8 + 11 + 14 + \ldots$

**Calculate the sum of the arithmetic series with each of the following descriptions. Use the formula $S_n = \frac{n(a_1 + a_n)}{2}$. (DOK1)**

9. $a_1 = 5, a_{20} = 81; n = 20$

10. $a_1 = \frac{3}{2}, a_{15} = \frac{45}{2}; n = 15$

**Calculate the sum of each of the following groups of numbers. (DOK 1)**

11. the first 75 positive odd integers

12. the first 59 multiples of 6

**Calculate the first differences and the second differences of the sequence of partial sums of each of the following arithmetic series. (DOK 1)**

13. $29 + 26 + 23 + 20 + 17$

14. $2 + 11 + 20 + 29 + 38$

**Determine if the series with each of the following sequences of first differences of partial sums is arithmetic. (DOK 1)**

15. 4, 16, 64, 256, 1024

16. 13, 20, 27, 34, 41



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**Find the formula for the sequences. Then find the 8th term in the sequence. (DOK 2)**

17. $-32 + -16 + -8 + -4 + -2 + \ldots$

18. $27+9+3+1+\frac{1}{3}+\frac{1}{9}+\ldots$

**Find the sum of the finite series when $n = 10$. (DOK 1)**

19. $27+9+3+1+\frac{1}{3}+\frac{1}{9}+\ldots$

20. $-32 + -16 + -8 + -4 + -2 + \ldots$

**Define the following sequences as a function and find the next term in the sequence. (DOK 2)**

21. $-1, 1, -1, 1, -1 \ldots$

22. $3, 8, 13, 18, 23 \ldots$

**Write each of the following recursive formulas in explicit form. (DOK 2)**

23. $f(1) = 1, f(n + 1) = f(n) + 2$

24. $f(1) = 7, f(n + 1) = f(n) + 5$

**Read the scenario below and answer the questions that follow. (DOK 3)**

Justin receives a bill from his Internet service provider. The first four months of service are charged according to the table below:

| | January | February | March | April |
|---|---|---|---|---|
| Hours | 0 | 10 | 5 | 25 |
| Charge | $4.95 | $14.45 | $9.70 | $28.70 |

25. Write a formula for the cost of $n$ hours of Internet service.

26. What is the greatest number of hours he can get on the Internet and still keep his bill under $20.00?

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